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Higher
Course
Specification

Higher Mathematics

Course code:	C847 76
Course assessment code:	X847 76
SCQF:	level 6 (24 SCQF credit points)
Valid from:	session 2023–24

This document provides detailed information about the course and course assessment to ensure consistent and transparent assessment year on year. It describes the structure of the course and the course assessment in terms of the skills, knowledge and understanding that are assessed.

This document is for teachers and lecturers and contains all the mandatory information you need to deliver the course.

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Contents

Course overview

The course consists of 24 SCQF credit points which includes time for preparation for course assessment. The notional length of time for candidates to complete the course is 160 hours.

The course assessment has two components.

Component	Marks	Duration
Question paper 1 (non-calculator)	55	1 hour and 15 minutes
Question paper 2	65	1 hour and 30 minutes

Recommended entry	Progression
Entry to this course is at the discretion of the centre. Candidates should have achieved the National 5 Mathematics course or equivalent qualifications and/or experience prior to starting this course.	♦ other qualifications in mathematics or related areas, for example Advanced Higher Mathematics, Advanced Higher Mathematics of Mechanics, Advanced Higher Statistics ♦ further study, employment and/or training

Conditions of award

The grade awarded is based on the total marks achieved across all course assessment components.

Achievement of this course gives automatic certification of the following Core Skill:

- Numeracy at SCQF level 6

Course rationale

National Courses reflect Curriculum for Excellence values, purposes and principles. They offer flexibility, provide time for learning, focus on skills and applying learning, and provide scope for personalisation and choice.

Every course provides opportunities for candidates to develop breadth, challenge and application. The focus and balance of assessment is tailored to each subject area.

Mathematics engages learners of all ages, interests and abilities. Learning mathematics develops logical reasoning, analysis, problem-solving skills, creativity, and the ability to think in abstract ways. It uses a universal language of numbers and symbols, which allows us to communicate ideas in a concise, unambiguous and rigorous way.

The course develops important mathematical techniques which are critical to successful progression beyond Higher level in Mathematics and many other curriculum areas. The skills, knowledge and understanding in the course also support learning in technology, health and wellbeing, science, and social studies.

Purpose and aims

Mathematics is important in everyday life. It helps us to make sense of the world we live in and to manage our lives.

Using mathematics enables us to model real-life situations and make connections and informed predictions. It equips us with the skills we need to interpret and analyse information, simplify and solve problems, assess risk and make informed decisions.

The course aims to:

- motivate and challenge candidates by enabling them to select and apply mathematical techniques in a variety of mathematical situations
- develop confidence in the subject and a positive attitude towards further study in mathematics and the use of mathematics in employment
- deliver in-depth study of mathematical concepts and the ways in which mathematics describes our world
- allow candidates to interpret, communicate and manage information in mathematical form, skills which are vital to scientific and technological research and development
- deepen candidates' skills in using mathematical language and exploring advanced mathematical ideas

Who is this course for?

This course is particularly suitable for candidates who:

- have demonstrated an aptitude for National 5 Mathematics
- are interested in developing mathematical techniques to use in further study or in the workplace

Course content

The Higher Mathematics course develops, deepens and extends the mathematical skills necessary at this level and beyond. Throughout this course, candidates acquire and apply operational skills necessary for developing mathematical ideas through symbolic representation and diagrams. They select and apply mathematical techniques and develop their understanding of the interdependencies within mathematics.

Candidates develop mathematical reasoning skills and gain experience in making informed decisions.

Skills, knowledge and understanding

Skills, knowledge and understanding for the course

The following provides a broad overview of the subject skills, knowledge and understanding developed in the course:

- understand and use a range of complex mathematical concepts and relationships
- select and apply operational skills in algebra, geometry, trigonometry, calculus and statistics within mathematical contexts
- select and apply skills in numeracy
- use mathematical reasoning skills to extract and interpret information and to use complex mathematical models
- use mathematical reasoning skills to think logically, provide justification or proof, and solve problems
- communicate mathematical information with complex features

Skills, knowledge and understanding for the course assessment

The following provides details of skills, knowledge and understanding sampled in the course assessment:

Algebraic and trigonometric skills	
Skills	**Explanation**
Manipulating algebraic expressions	♦ factorising a cubic or quartic polynomial expression ♦ simplifying a numerical expression using the laws of logarithms and exponents
Manipulating trigonometric expressions	♦ applying the addition formulae and/or double angle formulae ♦ applying trigonometric identities ♦ converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$, $k > 0$
Identifying and sketching related functions	♦ identifying a function from a graph, or sketching a function after a transformation of the form $kf(x)$, $f(kx)$, $f(x)+k$, $f(x+k)$ or a combination of these ♦ sketching $y = f'(x)$ given the graph of $y = f(x)$ ♦ sketching the inverse of a logarithmic or an exponential function ♦ completing the square in a quadratic expression where the coefficient of x^2 is non-unitary
Determining composite and inverse functions	♦ knowledge and use of the terms domain and range is expected ♦ determining a composite function given $f(x)$ and $g(x)$, where $f(x)$ and $g(x)$ can be trigonometric, logarithmic, exponential or algebraic functions ♦ determining $f^{-1}(x)$ of functions

Algebraic and trigonometric skills	
Skills	**Explanation**
Solving algebraic equations	♦ solving a cubic or quartic polynomial equation ♦ using the discriminant to find an unknown, given the nature of the roots of an equation ♦ solving quadratic inequalities, $ax^2+bx+c\geq 0$ (or ≤ 0) ♦ solving logarithmic and exponential equations ♦ using the laws of logarithms and exponents ♦ solving equations of the following forms for a and b, given two pairs of corresponding values of x and y: $\log y = b\log x + \log a,\ y = ax^b$ and $\log y = x\log b + \log a,\ y = ab^x$ ♦ using a straight-line graph to confirm relationships of the form $y = ax^b,\ y = ab^x$ ♦ mathematically modelling situations involving the logarithmic or exponential function ♦ finding the coordinates of the point(s) of intersection of a straight line and a curve or of two curves
Solving trigonometric equations	♦ solving trigonometric equations in degrees or radians, including those involving the wave function or trigonometric formulae or identities, in a given interval

Geometric skills	
Skills	**Explanation**
Determining vector connections	♦ determining the resultant of vector pathways in three dimensions ♦ working with collinearity ♦ determining the coordinates of an internal division point of a line
Working with vectors	♦ evaluating a scalar product given suitable information and determining the angle between two vectors ♦ applying properties of the scalar product ♦ using and finding unit vectors including $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as a basis

Calculus skills	
Skills	**Explanation**
Differentiating functions	♦ differentiating an algebraic function which is, or can be simplified to, an expression in powers of x ♦ differentiating $k \sin x$ and $k \cos x$ ♦ differentiating a composite function using the chain rule
Using differentiation to investigate the nature and properties of functions	♦ determining the equation of a tangent to a curve at a given point by differentiation ♦ determining where a function is strictly increasing or decreasing ♦ sketching the graph of an algebraic function by determining stationary points and their nature as well as intersections with the axes and behaviour of $f(x)$ for large positive and negative values of x

Calculus skills	
Skills	**Explanation**
Integrating functions	♦ integrating an algebraic function which is, or can be, simplified to an expression of powers of x ♦ integrating functions of the form $f(x)=(x+q)^n$, $n \neq -1$ ♦ integrating functions of the form $f(x)=p\cos x$ and $f(x)=p\sin x$ ♦ integrating functions of the form $f(x)=(px+q)^n$, $n \neq -1$ ♦ integrating functions of the form $f(x)=p\cos(qx+r)$ and $p\sin(qx+r)$ ♦ solving differential equations of the form $\frac{dy}{dx}=f(x)$
Using integration to calculate definite integrals	♦ calculating definite integrals of functions with limits which are integers, radians, surds or fractions
Applying differential calculus	♦ determining the optimal solution for a given problem ♦ determining the greatest and/or least values of a function on a closed interval ♦ solving problems using rate of change
Applying integral calculus	♦ finding the area between a curve and the x-axis ♦ finding the area between a straight line and a curve or two curves ♦ determining and using a function from a given rate of change and initial conditions

Algebraic and geometric skills	
Skills	**Explanation**
Applying algebraic skills to rectilinear shapes	♦ finding the equation of a line parallel to and a line perpendicular to a given line ♦ using $m = \tan\theta$ to calculate a gradient or angle ♦ using properties of medians, altitudes and perpendicular bisectors in problems involving the equation of a line and intersection of lines ♦ determining whether or not two lines are perpendicular
Applying algebraic skills to circles and graphs	♦ determining and using the equation of a circle ♦ using properties of tangency in the solution of a problem ♦ determining the intersection of circles or a line and a circle
Modelling situations using sequences	♦ determining a recurrence relation from given information and using it to calculate a required term ♦ finding and interpreting the limit of a sequence, where it exists

Reasoning skills	
Skills	**Explanation**
Interpreting a situation where mathematics can be used and identifying a strategy	♦ analysing a situation and identifying an appropriate use of mathematical skills
Explaining a solution and, where appropriate, relating it to context	♦ explaining why a particular solution is appropriate in a given context

Additional information
The following symbols, terms and sets may appear in the question papers. Candidates are expected to understand their use but they are not required to use them in their answers. ♦ the symbols: $\in$, $\notin$, $\{\ \}$ ♦ the terms: set, subset, empty set, member, element ♦ the conventions for representing sets, namely: — $\mathbb{N}$, the set of natural numbers, $\{1, 2, 3, \ldots\}$ — $\mathbb{W}$, the set of whole numbers, $\{0, 1, 2, 3, \ldots\}$ — $\mathbb{Z}$, the set of integers — $\mathbb{Q}$, the set of rational numbers — $\mathbb{R}$, the set of real numbers

Skills, knowledge and understanding included in the course are appropriate to the SCQF level of the course. The SCQF level descriptors give further information on characteristics and expected performance at each SCQF level, and can be found on the SCQF website.

Skills for learning, skills for life and skills for work

This course helps candidates to develop broad, generic skills. These skills are based on SQA's Skills Framework: Skills for Learning, Skills for Life and Skills for Work and draw from the following main skills areas:

2 Numeracy

2.1 Number processes
2.2 Money, time and measurement
2.3 Information handling

5 Thinking skills

5.3 Applying
5.4 Analysing and evaluating

You must build these skills into the course at an appropriate level, where there are suitable opportunities.

Course assessment

Course assessment is based on the information provided in this document.

The course assessment meets the key purposes and aims of the course by addressing:

- breadth — drawing on knowledge and skills from across the course
- challenge — requiring greater depth or extension of knowledge and/or skills
- application — requiring application of knowledge and/or skills in practical or theoretical contexts as appropriate

This enables candidates to:

- develop mathematical operational skills
- combine mathematical operational skills developed throughout the course
- develop mathematical reasoning skills
- apply skills, without the aid of a calculator, in order to demonstrate that they have an underlying grasp of mathematical concepts and processes

Course assessment structure

Question paper 1 (non-calculator) 55 marks

This question paper allows candidates to demonstrate the application of mathematical skills, knowledge and understanding from across the course. Candidates must not use a calculator.

This question paper gives candidates an opportunity to apply an understanding of the underlying processes involved in numerical, algebraic, geometric, trigonometric, calculus, and reasoning skills specified in the 'Skills, knowledge and understanding for the course assessment' section.

This question paper has 55 marks out of a total of 120 marks for the course assessment. It consists of short-answer and extended-response questions.

Setting, conducting and marking question paper 1 (non-calculator)

This question paper is set and marked by SQA, and conducted in centres under conditions specified for external examinations by SQA.

Candidates have 1 hour and 15 minutes to complete this question paper.

Question paper 2 65 marks

This question paper assesses mathematical skills. Candidates may use a calculator.

This question paper gives candidates an opportunity to apply numerical, algebraic, geometric, trigonometric, calculus, and reasoning skills specified in the ‘Skills, knowledge and understanding for the course assessment’ section.

Using a calculator can facilitate these skills and allow more opportunity for application and reasoning. Candidates typically use calculators where more complex calculations are required to solve problems.

This question paper has 65 marks out of a total of 120 marks for the course assessment. It consists of short-answer and extended-response questions.

Setting, conducting and marking question paper 2

This question paper is set and marked by SQA, and conducted in centres under conditions specified for external examinations by SQA.

Candidates have 1 hour and 30 minutes to complete this question paper.

Specimen question papers for Higher courses are published on SQA’s website. These illustrate the standard, structure and requirements of the question papers candidates sit. The specimen papers also include marking instructions.

Grading

Candidates’ overall grades are determined by their performance across the course assessment. The course assessment is graded A–D on the basis of the total mark for all course assessment components.

Grade description for C

For the award of grade C, candidates will typically have demonstrated successful performance in relation to the skills, knowledge and understanding for the course.

Grade description for A

For the award of grade A, candidates will typically have demonstrated a consistently high level of performance in relation to the skills, knowledge and understanding for the course.

Equality and inclusion

This course is designed to be as fair and as accessible as possible with no unnecessary barriers to learning or assessment.

For guidance on assessment arrangements for disabled candidates and/or those with additional support needs, please follow the link to the assessment arrangements web page: www.sqa.org.uk/assessmentarrangements.

Further information

The following reference documents provide useful information and background.

- Higher Mathematics subject page
- Assessment arrangements web page
- Building the Curriculum 3–5
- Guide to Assessment
- Guidance on conditions of assessment for coursework
- SQA Skills Framework: Skills for Learning, Skills for Life and Skills for Work
- Coursework Authenticity: A Guide for Teachers and Lecturers
- Educational Research Reports
- SQA Guidelines on e-assessment for Schools
- SQA e-assessment web page

The SCQF framework, level descriptors and handbook are available on the SCQF website.

Appendix 1: course support notes

Introduction

These support notes are not mandatory. They provide advice and guidance to teachers and lecturers on approaches to delivering the course. You should read these in conjunction with this course specification and the specimen question paper.

Approaches to learning and teaching

Approaches to learning and teaching should be engaging, with opportunities for personalisation and choice built in where possible.

A rich and supportive learning environment should be provided to enable candidates to achieve the best they can. This could include learning and teaching approaches such as:

- project-based tasks such as investigating the graphs of related functions, which could include using calculators or other technologies
- a mix of collaborative, co-operative or independent tasks, for example using differentiation to explore areas of science
- using materials available from service providers and authorities, for example working with a trigonometric model to predict the time of high tide in a harbour
- solving problems and thinking critically
- explaining thinking, and presenting strategies and solutions to others, such as discussing appropriate methods of solving trigonometric equations, perhaps using double angle formulae, and interpreting the solution set
- using questioning and discussion to encourage candidates to explain their thinking and to check their understanding of fundamental concepts
- making links in themes which cut across the curriculum to encourage transferability of skills, knowledge and understanding — including with technology, geography, sciences, social subjects and health and wellbeing — for example, using physics formulae and the application of calculus to the equations of motion under constant acceleration a, from initial speed u at position $x=0$ and time $t=0$ (for motion in a straight line):

  given $a=\frac{dv}{dt}$ integrate to get $v=u+at$ then note $v=\frac{ds}{dt}$ to get $s=ut+\frac{1}{2}at^2$

  sketch the graphs of a, v, and s versus t, and confirm the relationships using gradients and areas
- using technology, where appropriate, to extend experience and confidence

Developing mathematical skills is an active and productive process, building on candidates' current knowledge, understanding and capabilities. Existing knowledge should form the starting point for any learning and teaching situation, with new knowledge being linked to existing knowledge and built on. Presenting candidates with an investigative or practical task is a useful way of allowing them to appreciate how a new idea relates to their existing knowledge and understanding.

Questions can be used to ascertain a candidate's level of understanding and provide a basis for consolidation or remediation where necessary.

Examples of probing questions could include:

- How did you decide what to do?
- How did you approach exploring and solving this task or problem?
- Could this task or problem have been solved in a different way? If yes, what would you have done differently?

As candidates develop concepts in mathematics, they will benefit from continual reinforcement and consolidation to build a foundation for progression.

Throughout learning and teaching, candidates should be encouraged to:

- process numbers without using a calculator
- practise and apply the skills associated with mental calculations wherever possible
- develop and improve their skills in completing written and mental calculations in order to develop fluency and efficiency

The use of a calculator should complement these skills, not replace them.

Integrating skills

Integrating with other operational skills

Skills, knowledge and understanding may be integrated with other operational skills, for example:

- expressions could be combined with equations
- graphs of functions could be combined with equations
- differential calculus could be combined with optimisation
- integral calculus could be combined with area

Integrating with reasoning skills

Skills, knowledge and understanding may be integrated with reasoning skills, for example:

- algebraic or trigonometric expressions could be derived from a mathematical problem before being used in simplification
- the context of graphs could be discussed and interpretations made of related points
- vectors could be derived from a real-life situation
- equations can be interpreted/determined from geometrical diagrams
- recurrence relations can be determined from a real-life context
- problems of optimisation and area can be set from situations in science or technology
- the results of solving equations could be explained within a context

- a tangency problem could be set in a science context, such as an object being held in a circular motion and then released
- the value of definite integrals could be compared, particularly those in which the graphs cross the x-axis

Preparing for course assessment

The course assessment focuses on breadth, challenge and application. Candidates draw on and extend the skills they have learned during the course. These are assessed through two question papers: one non-calculator and a second paper in which a calculator may be used.

In preparation for the course assessment, candidates should be given the opportunity to:

- analyse a range of real-life problems and situations involving mathematics
- select and adapt appropriate mathematical skills
- apply mathematical skills with and without the aid of a calculator
- determine solutions
- explain solutions and/or relate them to context
- present mathematical information appropriately

The question papers assess a selection of knowledge and skills acquired during the course and provide opportunities for candidates to apply skills in a wide range of situations, some of which may be new.

Prior to the course assessment, candidates may benefit from responding to short-answer questions and extended-response questions.

Developing skills for learning, skills for life and skills for work

You should identify opportunities throughout the course for candidates to develop skills for learning, skills for life and skills for work.

Candidates should be aware of the skills they are developing and you can provide advice on opportunities to practise and improve them.

SQA does not formally assess skills for learning, skills for life and skills for work.

There may also be opportunities to develop additional skills depending on approaches being used to deliver the course in each centre. This is for individual teachers and lecturers to manage.

Some examples of potential opportunities to practise or improve these skills are provided in the following table.

SQA skills for learning, skills for life and skills for work framework definition	Suggested approaches for learning and teaching
Numeracy is the ability to use numbers to solve problems by counting, doing calculations, measuring, and understanding graphs and charts. It is also the ability to understand the results.	Candidates could be: ♦ given the opportunity to develop their numerical skills throughout the course, for example by using surds in differential and integral calculus, solving equations using exact trigonometric values, and simplifying expressions using the laws of logarithms ♦ given opportunities to use numbers to solve contextualised problems involving other STEM subjects ♦ encouraged to manage problems, tasks and case studies involving numeracy by analysing the context, carrying out calculations, drawing conclusions, making deductions and informed decisions
Applying is the ability to use existing information to solve a problem in a different context, and to plan, organise and complete a task.	Candidates could be: ♦ given the opportunity to apply the skills, knowledge and understanding they have developed to solve mathematical problems in a range of real-life contexts ♦ encouraged to think creatively to adapt strategies to suit the given problem or situation ♦ encouraged to show and explain their thinking to determine their level of understanding ♦ encouraged to think about how they are going to tackle problems or situations, decide which skills to use and then carry out the calculations necessary to complete the task, for example using the sine rule

SQA skills for learning, skills for life and skills for work framework definition	Suggested approaches for learning and teaching
Analysing and evaluating is the ability to identify and weigh up the features of a situation or issue and to use judgement to come to a conclusion. It includes reviewing and considering any potential solutions.	Candidates could be: ♦ given the opportunity to identify which real-life tasks or situations require the use of mathematics ♦ provided with opportunities to interpret the results of their calculations and to draw conclusions — conclusions drawn could be used to form the basis of making choices or decisions ♦ given the chance to identify and analyse situations involving mathematics which are of personal interest

During the course there are opportunities for candidates to develop their literacy skills and employability skills.

Literacy skills are particularly important as these skills allow candidates to access, engage in and understand their learning, and to communicate their thoughts, ideas and opinions. The course provides candidates with the opportunity to develop their literacy skills by analysing real-life contexts and communicating their thinking by presenting mathematical information in a variety of ways. This could include the use of numbers, formulae, diagrams, graphs, symbols and words.

Employability skills are the personal qualities, skills, knowledge, understanding and attitudes required in changing economic environments. The mathematical operational and reasoning skills developed in this course enable candidates to confidently respond to mathematical situations that can arise in the workplace. The course provides candidates with the opportunity to analyse a situation, decide which mathematical strategies to apply, work through those strategies effectively, and make informed decisions based on the results.

Additional skills for learning, skills for life and skills for work may also be developed during the course. These opportunities may vary and are at the discretion of the teacher or lecturer.

Appendix 2: skills, knowledge and understanding with suggested learning and teaching contexts

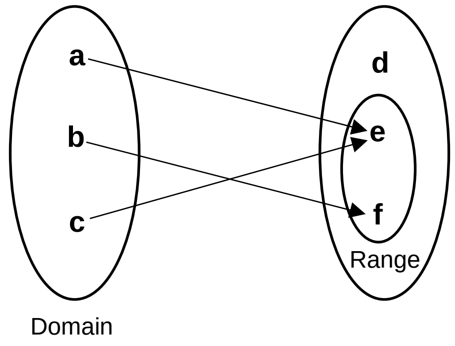
Examples of learning and teaching contexts that could be used for the course can be found below.

The first two columns are identical to the tables of 'Skills, knowledge and understanding for the course assessment' in this course specification.

The third column gives suggested learning and teaching contexts. These provide examples of where the skills could be used in individual activities or pieces of work.

Algebraic and trigonometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Manipulating algebraic expressions	♦ factorising a cubic or quartic polynomial expression	Teachers or lecturers could: ♦ demonstrate strategies for factorising polynomials, that is synthetic division, inspection, algebraic long division (From previous learning, candidates should be able to factorise quadratic expressions. They can link these solution(s) to the graph of a function. Factorising polynomials beyond degree two allows extension of this concept.)
	♦ simplifying a numerical expression using the laws of logarithms and exponents	♦ link the logarithmic scale to science applications, for example decibel scale for sound, Richter scale of earthquake magnitude, astronomical scale of stellar brightness, acidity and pH in chemistry and biology (Note link between scientific notation and logs to base 10.)

Algebraic and trigonometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Manipulating trigonometric expressions	♦ applying the addition formulae and/or double angle formulae ♦ applying trigonometric identities ♦ converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$, $k > 0$	Teachers or lecturers could: ♦ show candidates how formulae for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$ can be used to prove formulae for $\sin 2\alpha$, $\cos 2\alpha$ and $\tan(\alpha+\beta)$ ♦ emphasise the distinction between $\sin x°$ and $\sin x$ (degrees and radians) ♦ give candidates practice in applying the standard formulae, for example expand $\sin 3x$ or $\cos 4x$ ♦ set candidates geometric problems which require the use of addition or double angle formulae
Identifying and sketching related functions	♦ identifying a function from a graph, or sketching a function after a transformation of the form $kf(x)$, $f(kx)$, $f(x)+k$, $f(x+k)$ or a combination of these ♦ sketching $y = f'(x)$ given the graph of $y = f(x)$ ♦ sketching the inverse of a logarithmic or an exponential function ♦ completing the square in a quadratic expression where the coefficient of x^2 is non-unitary	Candidates could use graphic calculators to explore various transformations of functions. Candidates should be able to: ♦ recognise a function from its graph ♦ interpret formulae or equations for maximum and minimum values and identify when they occur

Algebraic and trigonometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Determining composite and inverse functions	♦ knowledge and use of the terms domain and range are expected ♦ determining a composite function given $f(x)$ and $g(x)$, where $f(x)$ and $g(x)$ can be trigonometric, logarithmic, exponential or algebraic functions ♦ determining $f^{-1}(x)$ of functions	The use of balloon or arrow diagrams and Cartesian graphs can help reinforce the definition of function, domain, and range.  Arrow diagrams can also be used to demonstrate that the composite function $f(g(x))$ may not always exist. Diagrams or graphs can also be used to establish whether or not a given function has an inverse. Candidates should be aware that $f(g(x))=x$ implies $f(x)$ and $g(x)$ are inverses.

Algebraic and trigonometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Solving algebraic equations	♦ solving a cubic or quartic polynomial equation ♦ using the discriminant to find an unknown, given the nature of the roots of an equation ♦ solving quadratic inequalities, $ax^2+bx+c\geq 0$ (or ≤ 0) ♦ solving logarithmic and exponential equations ♦ using the laws of logarithms and exponents ♦ solving equations of the following forms for a and b, given two pairs of corresponding values of x and y: $\log y = b\log x + \log a,\ y = ax^b$ and $\log y = x\log b + \log a,\ y = ab^x$ ♦ using a straight-line graph to confirm relationships of the form $y = ax^b,\ y = ab^x$ ♦ mathematically modelling situations involving the logarithmic or exponential function ♦ finding the coordinates of the point(s) of intersection of a straight line and a curve or of two curves	Teachers or lecturers could: ♦ demonstrate when expressions are not polynomial (negative or fractional powers) ♦ explain that a repeated root is also a stationary point ♦ emphasise the meaning of solving $f(x)=g(x)$ ♦ introduce the Remainder Theorem by: — demonstrating how, for a polynomial equation, this leads to the fact that $f(x)=0$, if $(x-h)$ is a factor of $f(x)$ and h is a root of the equation and vice versa — explaining that communication should include a statement such as 'since $f(h)=0$' or 'since remainder is 0' — using divisors and/or factors of the form $(ax-b)$ ♦ link the solutions of algebraic equations to a graph of function(s), where possible, and encourage candidates to make this connection — candidates could use graphic calculators or refer to diagrams in the question or sketch diagrams to check their solutions ♦ use real-life contexts involving logarithmic and exponential characteristics, for example rate of growth of bacteria, calculations of money earned at various interest rates over time, decay rates of radioactive materials

Algebraic and trigonometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Solving trigonometric equations	♦ solving trigonometric equations in degrees or radians, including those involving the wave function or trigonometric formulae or identities, in a given interval	Teachers and lecturers could: ♦ use real-life contexts, for example: — A possible application is the refraction of a thin light beam passing from air into glass. Its direction of travel is bent towards the line normal to the surface, according to Snell's law. ♦ demonstrate how trigonometric equations can be solved graphically ♦ explain that in the absence of a degree symbol, candidates should use radians in solutions, for example $0 \leq x \leq \pi$

Geometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Determining vector connections	♦ determining the resultant of vector pathways in three dimensions ♦ working with collinearity ♦ determining the coordinates of an internal division point of a line	Candidates should: ♦ work with vectors in both two and three dimensions ♦ mention 'parallel vectors' and 'common point' in their solutions to show collinearity ♦ be able to distinguish between coordinate and component forms
Working with vectors	♦ evaluating a scalar product given suitable information and determining the angle between two vectors ♦ applying properties of the scalar product ♦ using and finding unit vectors including $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as a basis	Teachers and lecturers could: ♦ introduce candidates to the zero vector, for example through its broader application: — They could sketch a vector diagram of the three forces on a kite, when stationary: its weight, force from the wind (assume normal to centre of kite inclined facing the breeze) and its tethering string. These must sum to zero. ♦ explain the perpendicular and distributive properties of vectors, for example if $\lvert\mathbf{a}\rvert, \lvert\mathbf{b}\rvert \neq 0$ then $\mathbf{a}\cdot\mathbf{b}=0$ if and only if the directions of $\mathbf{a}$ and $\mathbf{b}$ are at right angles

Calculus skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Differentiating functions	♦ differentiating an algebraic function which is, or can be simplified to, an expression in powers of x ♦ differentiating $k \sin x$ and $k \cos x$ ♦ differentiating a composite function using the chain rule	Teachers and lecturers could use examples from science and terms associated with rates of change, for example acceleration, velocity.
Using differentiation to investigate the nature and properties of functions	♦ determining the equation of a tangent to a curve at a given point by differentiation ♦ determining where a function is strictly increasing or decreasing ♦ sketching the graph of an algebraic function by determining stationary points and their nature as well as intersections with the axes and behaviour of $f(x)$ for large positive and negative values of x	Candidates should know: ♦ that the gradient of a curve at a point is defined to be the gradient of the tangent to the curve at that point ♦ when a function is either strictly increasing, decreasing or has a stationary value, and the conditions for these Candidates can use the second derivative or a detailed nature table. Stationary points should include horizontal points of inflexion.

Calculus skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Integrating functions	♦ integrating an algebraic function which is, or can be, simplified to an expression of powers of x ♦ integrating functions of the form $f(x)=(x+q)^n$, $n \neq -1$ ♦ integrating functions of the form $f(x)=p\cos x$ and $f(x)=p\sin x$ ♦ integrating functions of the form $f(x)=(px+q)^n$, $n \neq -1$ ♦ integrating functions of the form $f(x)=p\cos(qx+r)$ and $p\sin(qx+r)$ ♦ solving differential equations of the form $\dfrac{dy}{dx}=f(x)$	Candidates should know: ♦ the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve ♦ that if $f(x)=F'(x)$ then $\int_a^b f(x)\,dx = F(b)-F(a)$ and $\int f(x)\,dx = F(x)+C$ where C is the constant of integration Teachers and lecturers could: ♦ introduce integration as the process of finding anti-derivatives ♦ demonstrate how to integrate $\cos^2 x$ and $\sin^2 x$ using $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
Using integration to calculate definite integrals	♦ calculating definite integrals of functions with limits which are integers, radians, surds or fractions	

Calculus skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Applying differential calculus	♦ determining the optimal solution for a given problem ♦ determining the greatest and/or least values of a function on a closed interval ♦ solving problems using rate of change	Teachers and lecturers could: ♦ apply maximum and/or minimum problems in real-life contexts, for example minimum amount of card for creating a box, maximum output from machines ♦ link rate of change to science contexts, for example optimisation in science: An aeroplane cruising at speed v at a steady height has to use power to push air downwards to counter the force of gravity and to overcome air resistance to sustain its speed. The energy cost per km of travel is given approximately by: $E = Av^2 + Bv^2$. (A and B depend on the size and weight of the plane.) At the optimum speed $\frac{dE}{dv} = 0$, thus get an expression for v_{opt} in terms of A and B.

Calculus skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Applying integral calculus	♦ finding the area between a curve and the x-axis ♦ finding the area between a straight line and a curve or two curves ♦ determining and using a function from a given rate of change and initial conditions	Teachers and lecturers could demonstrate how to: ♦ use graphical calculators as part of an investigative approach ♦ calculate the area between curves by subtracting individual areas, using diagrams or graphing packages ♦ reduce the area to be determined to smaller components to estimate a segment of area between the curve and x-axis and then use the area formulae (triangle or rectangle) A practical application of the integral of $\frac{1}{x^2}$ is to calculate the energy required to lift an object from the earth's surface into space. The work energy required is $E = \int F dr$, where F is the force due to the earth's gravity and r is the distance from the centre of the earth. For a 1 kg object $E = -\int \left(\frac{GM}{r^2} \right) dr$, where M is the mass of the earth and G is the universal gravitational constant. $GM = 4 \cdot 0 \times 10^{14}$ m^3 s^{-2} The integration extends from $r = 6 \cdot 4 \times 10^6$ m (the radius of the earth) to infinity.

Algebraic and geometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Applying algebraic skills to rectilinear shapes	♦ finding the equation of a line parallel to and a line perpendicular to a given line ♦ using $m = \tan\theta$ to calculate a gradient or angle ♦ using properties of medians, altitudes and perpendicular bisectors in problems involving the equation of a line and intersection of lines ♦ determining whether or not two lines are perpendicular	Teachers and lecturers could: ♦ emphasise the 'gradient properties' of $m_1 = m_2$ and $m_1 m_2 = -1$ ♦ use practical contexts for triangle work, where possible ♦ emphasise differences in median, altitude, etc ♦ investigate properties and intersections Candidates should: ♦ avoid approximating gradients to decimals ♦ have knowledge of the basic properties of triangles and quadrilaterals ♦ include the phrases 'parallel' and 'common point' in their answers to show collinearity, for example since $m_{AB} = m_{BC} \Rightarrow$ AB and BC are parallel, and B is a common point ♦ understand terms such as orthocentre, circumcentre and concurrency

Algebraic and geometric skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Applying algebraic skills to circles and graphs	♦ determining and using the equation of a circle ♦ using properties of tangency in the solution of a problem ♦ determining the intersection of circles or a line and a circle	Teachers and lecturers could: ♦ develop the equation of a circle (centre the origin) from Pythagoras, and extend this to a circle with centre (a,b) or relate to transformations ♦ link the properties of tangency with the application of the discriminant ♦ make candidates aware of different ways in which more than one circle can be positioned, for example intersecting at one, two, or no points, sharing the same centre (concentric), one circle inside another ♦ give candidates practice in applying knowledge of geometric properties of circles to find related points (for example the stepping-out method) — solutions should not be obtained from scale drawings
Modelling situations using sequences	♦ determining a recurrence relation from given information and using it to calculate a required term ♦ finding and interpreting the limit of a sequence, where it exists	Teachers and lecturers could use examples from a real-life context, for example a situation where the concentration of chemicals or medicines is important.

Reasoning skills		
Skills	**Explanation**	**Suggested learning and teaching contexts**
Interpreting a situation where mathematics can be used and identifying a strategy	♦ analysing a situation and identifying an appropriate use of mathematical skills	Teachers and lecturers could give candidates a mathematical or real-life problem in which some analysis is required. Candidates should choose an appropriate strategy and use mathematics to solve the problem.
Explaining a solution and, where appropriate, relating it to context	♦ explaining why a particular solution is appropriate in a given context	Candidates should use everyday language to give meaning to the determined solution.

Appendix 3: question paper brief

The course assessment consists of two question papers which assess the:

- development of mathematical operational skills
- combination of mathematical operational skills
- development of mathematical reasoning skills
- application of skills, without the aid of a calculator, in order to demonstrate candidates' underlying grasp of mathematical concepts and processes

The question papers sample the 'Skills, knowledge and understanding' section of the course specification.

This sample draws on all of the skills, knowledge and understanding from each of the following areas:

- numerical skills
- algebraic skills
- geometric skills
- trigonometric skills
- calculus skills
- reasoning skills

Command words are the verbs or verbal phrases used in questions and tasks which ask candidates to demonstrate specific skills, knowledge or understanding. For examples of some of the command words used in this assessment, refer to the past papers and specimen question paper.

The course assessment consists of two question papers:

	Paper 1 (non-calculator)	Paper 2
Time	1 hour and 15 minutes	1 hour and 30 minutes
Marks	55	65
Skills	This question paper gives candidates an opportunity to apply numerical, algebraic, geometric, trigonometric, calculus and reasoning skills, **without the aid of a calculator**. Candidates are required to show an understanding of underlying processes and the ability to use skills within mathematical contexts in cases where a calculator may compromise the assessment of this understanding.	This question paper gives candidates an opportunity to apply numerical, algebraic, geometric, trigonometric, calculus and reasoning skills. These skills may be facilitated by using a calculator, as this allows more opportunity for application and reasoning.
Percentage of marks across the papers	Approximately 30–45% of the overall marks relate to algebra. Approximately 15–35% of the overall marks relate to geometry. Approximately 15–40% of the overall marks relate to calculus. Approximately 10–25% of the overall marks relate to trigonometry.	
Type of question	Short-answer and extended-response questions	
Type of question paper	Semi-structured question papers: separate question paper and answer booklet. The answer booklet is structured with spaces for answers.	
Proportion of level ‘C’ questions	Some questions use a stepped approach to ensure that there are opportunities for candidates to demonstrate their abilities beyond level ‘C’. Approximately 65% of marks are available for level ‘C’ responses.	
Balance of skills	Operational and reasoning skills are assessed in both question papers. Some questions assess only operational skills (approximately 65% of the marks), but other questions assess operational and reasoning skills (approximately 35% of the marks).	

Administrative information

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History of changes

Version	Description of change	Date
2.0	Course support notes; skills, knowledge and understanding with suggested learning and teaching contexts; and question paper brief added as appendices.	May 2018
3.0	Question paper duration and marks amended.	May 2023

Note: you are advised to check SQA's website to ensure you are using the most up-to-date version of this document.