Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2023

Mathematics

Assessment Unit AS 2

assessing

Applied Mathematics

[SMT21]

TUESDAY 30 MAY, AFTERNOON

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A: Mechanics

AVAILABLE MARKS

1 **(i)** speed = $\sqrt{(6)^2 + (-8)^2}$ — M1

$= 10\ \text{ms}^{-1}$ — W1

(ii)

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$40\mathbf{i} + 8\mathbf{j} = 4(6\mathbf{i} - 8\mathbf{j}) + \frac{1}{2}\mathbf{a}\,(4)^2$$ — M1

$$40\mathbf{i} + 8\mathbf{j} = 24\mathbf{i} - 32\mathbf{j} + 8\mathbf{a}$$

$$8\mathbf{a} = 16\mathbf{i} + 40\mathbf{j}$$

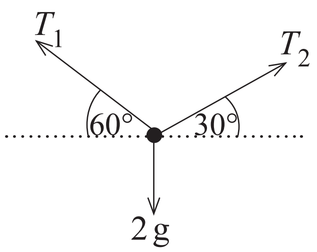
$$\mathbf{a} = (2\mathbf{i} + 5\mathbf{j})\ \text{ms}^{-2}$$ — W1

Using $\mathbf{F} = m\mathbf{a}$

$$\mathbf{F} = 5(2\mathbf{i} + 5\mathbf{j})$$

$$\mathbf{F} = (10\mathbf{i} + 25\mathbf{j})\ \text{N}$$ — MW1 — 5

2 **(i)**



MW2

(ii) Resolving forces vertically — M1

$T_1 \sin 60° + T_2 \sin 30° = 2\,g$ — W1

$$\frac{\sqrt{3}}{2}T_1 + \frac{1}{2}T_2 = 2\,g$$

$$\sqrt{3}T_1 + T_2 = 4\,g \qquad (1)$$

Resolving forces horizontally:

$T_1 \cos 60° = T_2 \cos 30°$ — MW1

$T_1 = \sqrt{3}\,T_2 \qquad (2)$

Attempting to solve simultaneous equations — M1

$$\sqrt{3}\,(\sqrt{3}\,T_2) + T_2 = 4\,g$$

$$T_2 = 9.8\text{N}$$ — W1

$$\therefore T_1 = \sqrt{3}\,T_2$$

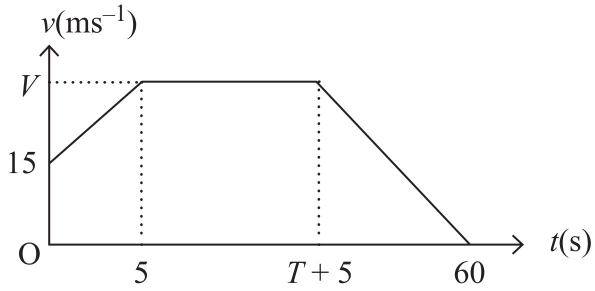
$$\therefore T_1 = 16.974\ldots$$

$$\therefore T_1 = 17.0\ \text{N}$$ — W1

(iii) Strings have negligible mass. — MW1

(iv) The tensions increase. — MW1 — 10

3 **(i)**



	MW2	
(ii) $a = \dfrac{V - u}{t}$	M1	
$2 = \dfrac{V - 15}{5}$		
$V = 25\ \text{ms}^{-1}$	W1	
Total distance travelled = area under the graph	M1	
$1000 = \frac{1}{2}(15 + 25) \times 5 + 25T + \frac{1}{2}(60 - (T + 5)) \times 25$	MW3	
$1000 = 787.5 + 12.5T$		
$T = 17$	W1	9
4 **(i)** Resolving forces acting on Q and using $F = ma$	M1	
$5\,\text{g} - T = 5a$	W1	
Resolving forces acting on P perpendicular to the plane:		
$R = 4\,\text{g}\cos\alpha$	MW1	
$R = \dfrac{16\,\text{g}}{5}$ N (31.36N)		
Resolving forces acting on P parallel to the plane and using $F = ma$		
$T - Fr - 4\,\text{g}\sin\alpha = 4a$	M1 W1	
$T - \mu R - 4\,\text{g}\sin\alpha = 4a$	MW1	
$T - 0.8\,\text{g} - 2.4\,\text{g} = 4a$		
Trying to solve simultaneous equations		
$5\text{g} - 5a - 3.2\,\text{g} = 4a$	M1	
$9a = 1.8\,\text{g}$		
$a = 1.96\ \text{ms}^{-2}$	W1	
(ii) Finding a new acceleration	M1	
$-4\,\text{g}\sin\alpha - \mu R = 4a$		
$a = -7.84\ \text{ms}^{-2}$	W1	
Using $v = u + at$		
$0 = 2.94 - 7.84t$		
$t = 0.375$ s	MW1	11

AVAILABLE MARKS

Section B: Statistics

				AVAILABLE MARKS
5	(i)	The population is the complete set of objects of interest under a statistical hypothesis, and a sample is a subset of the population.	MW2	
	(ii)	There are distinct sections of the population which are in different proportions and a simple random sample may not reflect this.	MW2	
	(iii)	Stratified sampling.	MW1	5
6	(i)	Frequencies: 27, 54, 45, 18, 6	MW1	
		$\bar{x} = \dfrac{27 \times 15 + 54 \times 45 + 45 \times 75 + 18 \times 105 + 6 \times 150}{27 + 54 + 45 + 18 + 6}$	M1	
		$= 60$ minutes $= 1$ hour	W1	
	(ii)	$\Sigma fx^2 = 27 \times 15^2 + 54 \times 45^2 + 45 \times 75^2 + 18 \times 105^2 + 6 \times 150^2$ $= 702\,000$	MW1	
		$\sigma^2 = \dfrac{702000}{150} - 60^2$	M1	
		$= 1080$ minutes^2	W1	
	(iii)	$\sigma = \sqrt{1080} = 32.9$ minutes	MW1	
		The standard deviation is smaller for the second Saturday than the first, so there was less variability in the times spent by the vehicles in the car park on the second Saturday than the first.	MW2	9
7	(i)	$P(B_1 \cap \bar{B}_2) + P(\bar{B}_1 \cap B_2) = 2 \times 0.45 \times 0.55$	M1	
		$= 0.495$	W1	
	(ii)	The binomial distribution is an appropriate model because:	MW1	
		• there are two mutually exclusive, exhaustive outcomes (make offer, don't make offer); • the number of clients is fixed at 20; • clients make offers, or not, independently of each other; • there is a fixed probability of success.	MW3	
	(iii)	Let X = No. of offers received after viewing the property. $X \sim \text{Bin}(20, 0.45)$	M1	
		$P(X = 10) = {}^{20}C_{10}\,(0.45)^{10}\,(0.55)^{10}$	MW1	
		$= 0.159$	W1	
	(iv)	$P(X < 5) = P(X \leq 4)$	M1	
		$= {}^{20}C_0\,(0.45)^0\,(0.55)^{20} + {}^{20}C_1\,(0.45)^1\,(0.55)^{19} + {}^{20}C_2\,(0.45)^2\,(0.55)^{18}$ $+ {}^{20}C_3\,(0.45)^3\,(0.55)^{17} + {}^{20}C_4\,(0.45)^4\,(0.55)^{16}$	MW1	
		$= 0.0189$	W1	12

			AVAILABLE MARKS
8	**(i)** Let $x = P(X)$ so $P(Y) = 3x$	MW1	
	$P(\overline{X} \cap \overline{Y}) = 1 - P(X \cup Y)$	M1	
	$= 1 - [P(X) + P(Y)]$	M1	
	$= 1 - [x + 3x]$		
	$x = 1 - 4x$	MW1	
	$x = \frac{1}{5}$	W1	
	(ii) $P(X) + P(Y) = \frac{1}{5} + \frac{3}{5} = \frac{4}{5} \neq 1$	MW1	
	(iii) Since X and Y are mutually exclusive, $P(X \cap Y) = 0$ but $P(X)$ and $P(Y)$ are both non-zero, so X and Y cannot be independent.	MW3	9
		Total	**70**