Rewarding Learning

ADVANCED
General Certificate of Education
2022

Centre Number

Candidate Number

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

AMT11

[AMT11]
MONDAY 6 JUNE, MORNING

TIME

2 hours 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eleven** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit.**
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

12948.06 **R**

1 The 4th and 5th terms of a geometric progression are $-40\frac{1}{2}$ and $30\frac{3}{8}$ respectively.

(i) Find the common ratio. [3]

(ii) Find the value of the first term. [2]

(iii) Find the sum to infinity for this series. [2]

2 Differentiate the following with respect to x, simplifying your answer where possible.

(i) $y = 3x(2x + 5)^4$ [4]

(ii) $y = \dfrac{x - 7}{5x^2 + 4}$ [4]

(iii) $y = (1 + \ln 3x)^5$ [4]

[Turn over

3 (a) (i) Prove that

$$\sec 2\theta \cos \theta \equiv \frac{\cos \theta}{2\cos^2 \theta - 1}$$

[3]

(ii) Hence find the exact solutions of

$$\sec 2\theta \cos \theta = 1 \qquad 0 \leqslant \theta \leqslant 2\pi$$ [5]

(b) A new ice cream shop is opening and a designer has created a logo as shown in **Fig. 1** below.

The logo consists of two congruent triangles, AOD and BOC, together with a sector of a circle centred at O.

$AD = BC = 6\sqrt{21}$ cm, $OC = OD = 6$ cm, $AB = 60$ cm

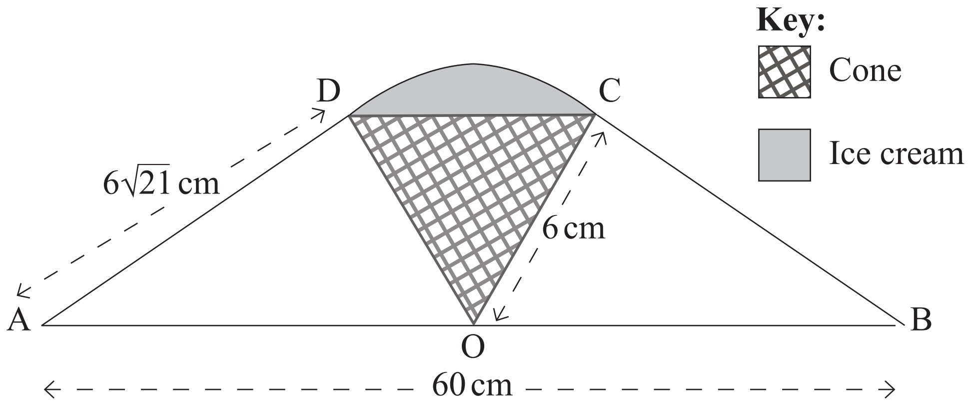

Fig. 1

(i) Find the exact angle DOC in radians. [4]

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(ii) Find the exact area represented by the ice cream as shown in **Fig. 1** [6]

4 The graph of the function $y = \mathrm{f}(x)$ is sketched in **Fig. 2** below.

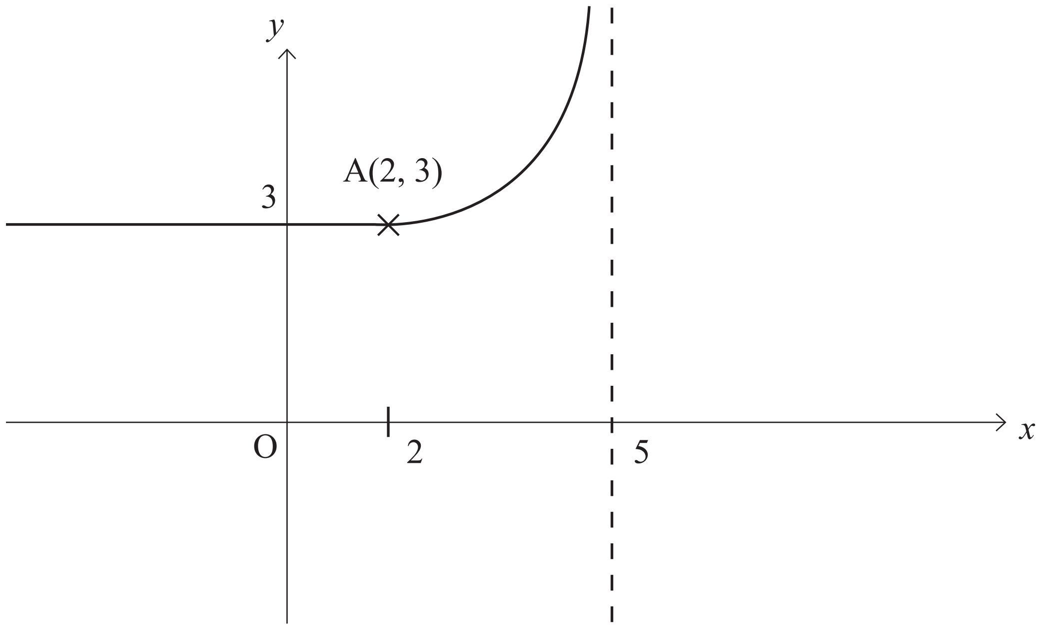
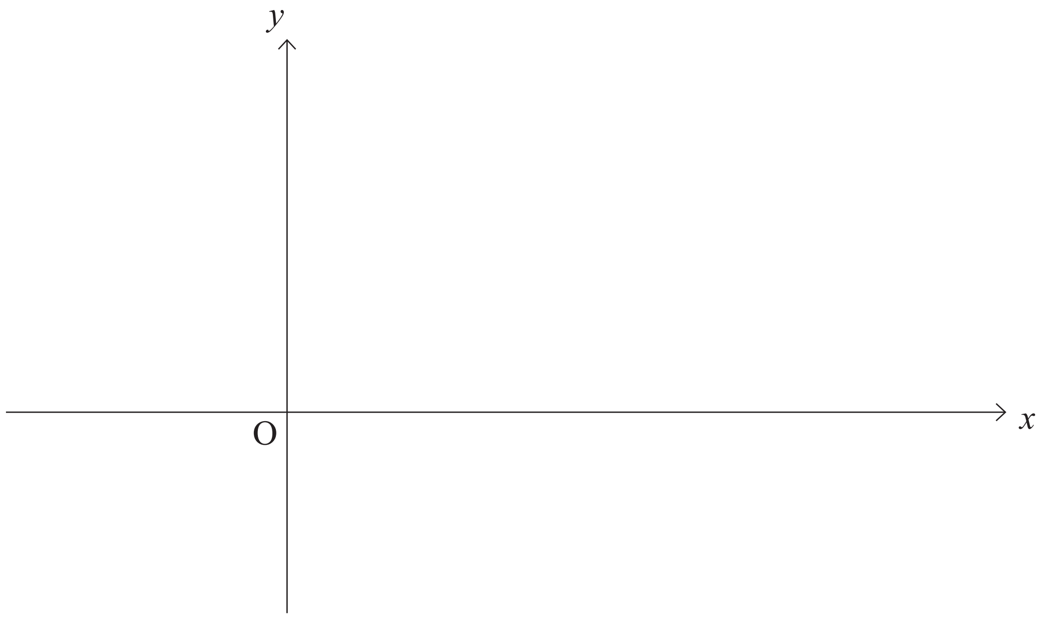


Fig. 2

(i) On the axes below, sketch the graph of

$$y = \mathrm{f}(x + 2) + 1$$

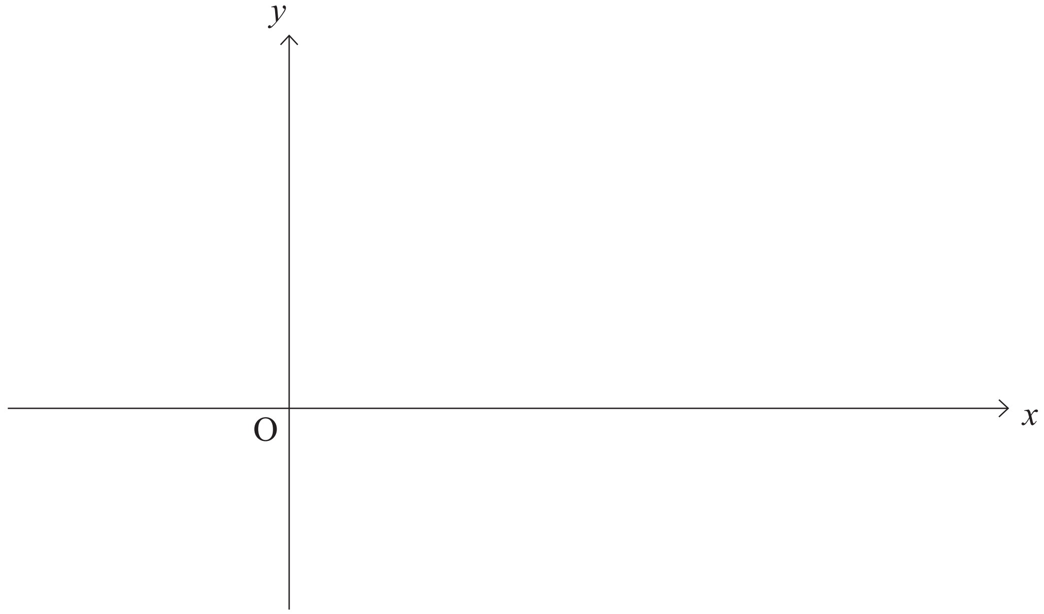
Clearly identify the image of A and the location of the asymptote. [2]



(ii) On the axes below, sketch the graph of

$$y = 2\,\mathrm{f}(x) + 1$$

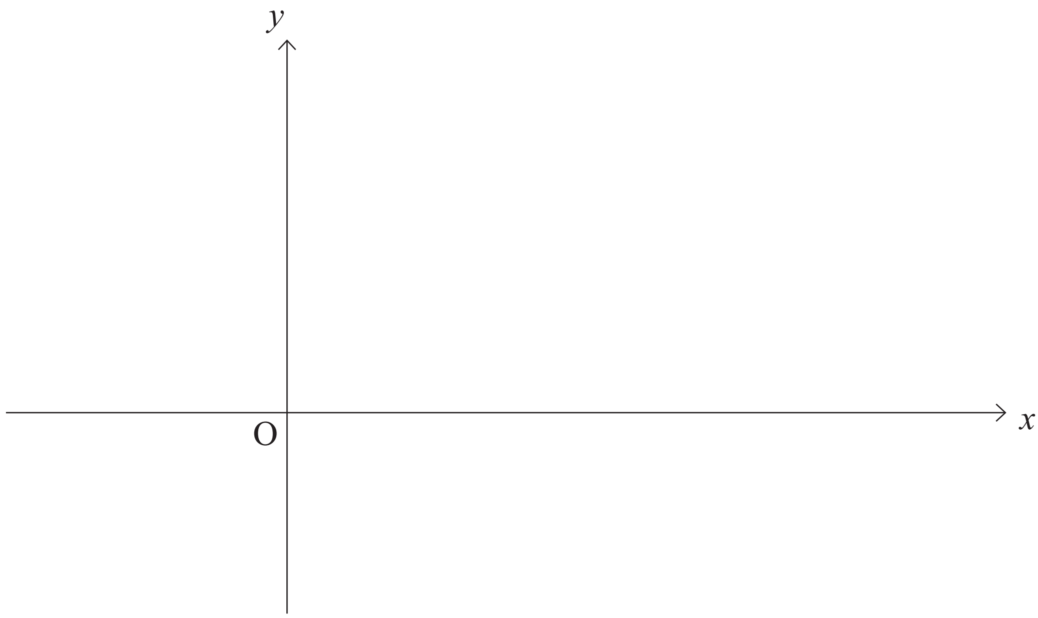
Clearly identify the image of A and the location of the asymptote. [3]



(iii) On the axes below, sketch the graph of

$$y = \mathrm{f}\left(\frac{x}{2} - 1\right)$$

Clearly identify the image of A and the location of the asymptote. [3]



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(Questions continue overleaf)

5 The function $f(x)$ is defined as

$$f(x) = -x^2 + 3x + 10, \quad x \in \mathbb{R}$$

(i) Express $f(x)$ in the form $-(x - a)^2 + b$ [3]

(ii) Hence state the range of the function $f(x)$ [1]

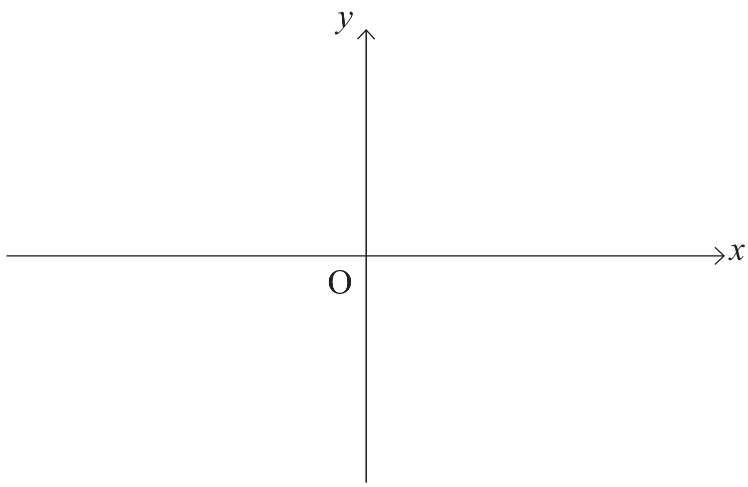
(iii) State the domain for which $f(x)$ is a decreasing function. [1]

(iv) Given

$$f(x) = -x^2 + 3x + 10, \quad x \in \mathbb{R} \quad \text{and}$$

$$g(x) = |x|, \quad x \in \mathbb{R}$$

sketch the composite function gf(x) on the axes below, showing clearly any key points. [5]



6 **(i)** Write the following expression in partial fractions

$$\frac{x+5}{(3-x)(1+x)^2}$$ [7]

(ii) Hence, using the binomial theorem, expand

$$\frac{x+5}{(3-x)(1+x)^2}$$

in ascending powers of x, up to and including the term in x^2 [10]

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(Questions continue overleaf)

7 Integrate the following with respect to x

(i) $\int \sin^2 x \ \, dx$ [5]

(ii) $\int xe^{3x}\ dx$ [7]

(iii) $\int \frac{x^2}{\sqrt{x-2}}\,dx$ using the substitution $u^2 = x - 2$ [7]

8 The ambient temperature of a restaurant remains constant at 20°C.

θ is the temperature of food t minutes after leaving the kitchen and the rate of change in temperature can be modelled by the following differential equation

$$\frac{d\theta}{dt} = -k(\theta - 20)$$

(i) Given that food leaves the kitchen with a temperature of 140°C, find θ in terms of k and t. [7]

A food critic, visiting the restaurant, notes that it takes 2 minutes for her food to arrive and records the temperature of her meal as 75°C.

Andrew is a chef in the restaurant. He believes that food from his kitchen should be served to customers at a temperature of at least 60°C.

(ii) Find the time in which a waiter must have delivered the food in order to meet the chef's requirement. Give your answer in minutes and seconds to the nearest second. [5]

9 An implicit function is defined as

$$x^2y + 2x^2 = 10y + 4$$

Find the equation of the normal to the curve at the point $(\sqrt{2}, 0)$.

Leave your answer in the form $y = -\sqrt{a}x + b$ where a and b are integer values. [10]

10 **Fig. 3** below shows the graph of the parametric equations

$$x = \cos\theta, \qquad y = \sin 2\theta, \qquad 0 \leqslant \theta \leqslant 2\pi$$

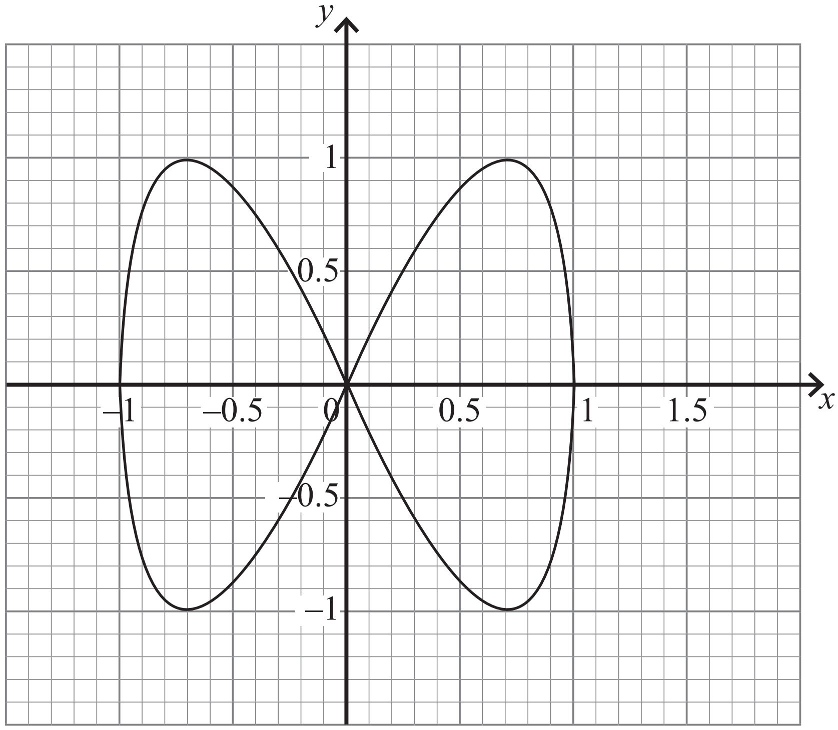

Fig. 3

(i) Show that $\dfrac{dy}{dx} = -\dfrac{2\cos 2\theta}{\sin\theta}$ [4]

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(ii) Determine the values of θ for the four stationary points. [5]

(iii) Hence find the exact Cartesian coordinates of the four stationary points. [4]

(iv) Find the Cartesian equation of the curve shown in **Fig. 3** [4]

In **Fig. 4** below, the area bounded by this curve, the x-axis and the line $x = 0.5$ is shaded.

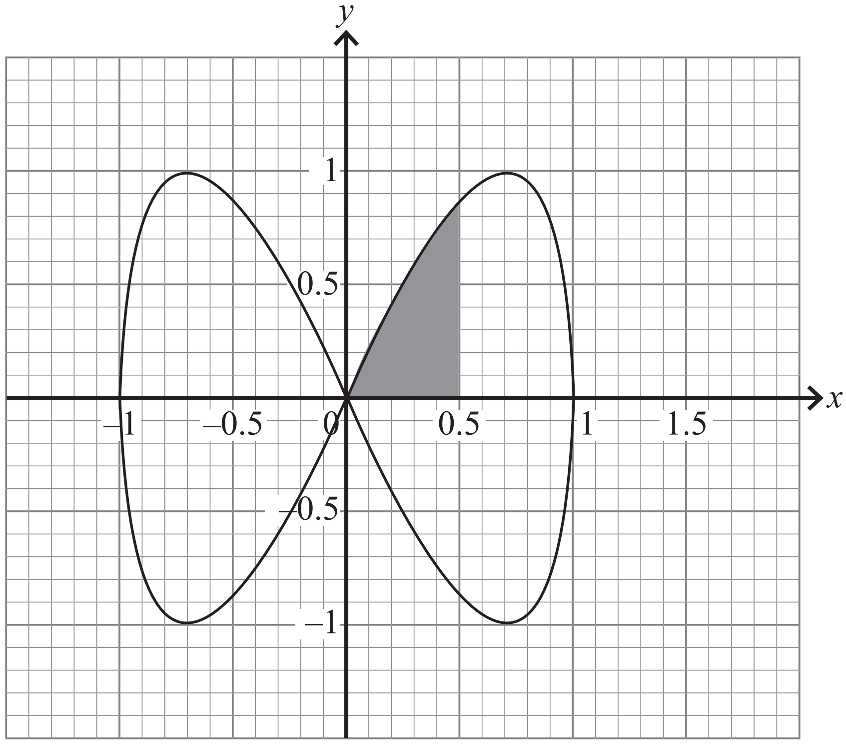

Fig. 4

This area is rotated through 2π radians about the x-axis to form a paperweight.

(v) Using your answer to **(iv)**, find the volume of material needed to create the paperweight. [6]

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11 The curves of two functions are shown in **Fig. 5** below.

$$g(x) = \sqrt{3}x^{\frac{1}{2}} \qquad h(x) = \frac{1}{2}x^2 + x^5$$

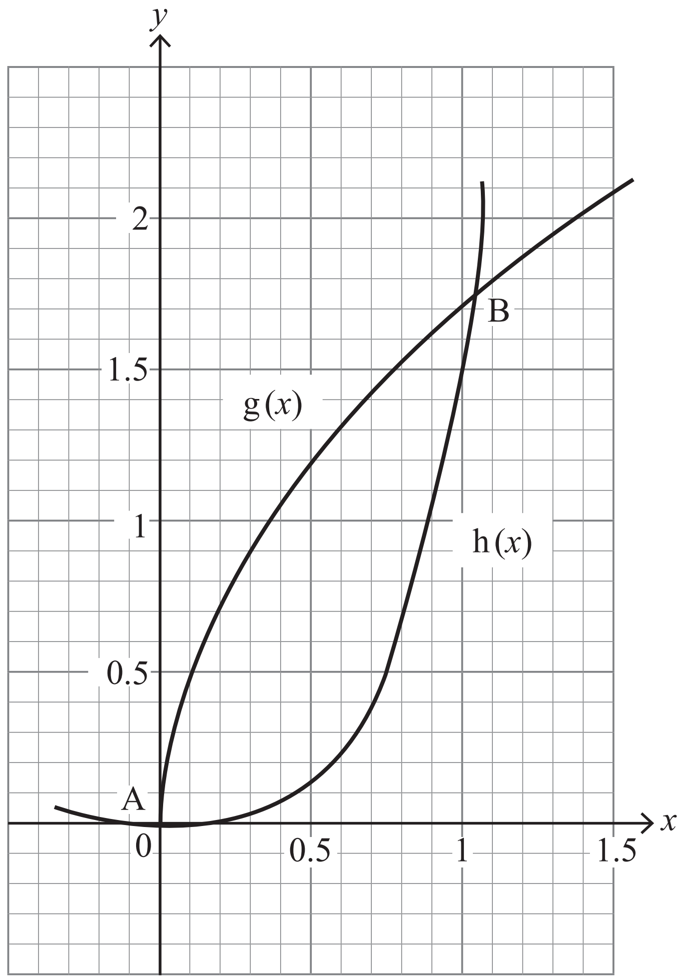

Fig. 5

The graphs of the functions intersect at points A and B.

Point A is at the origin and point B has an x-coordinate close to 1

(i) Use the Newton-Raphson method once, with $x_0 = 1$, to find a better approximation for the x-coordinate of B.

Give your answer correct to three decimal places. [7]

(ii) Find the area enclosed between these two curves, using the approximation you have found in **(i)**.

Give your answer correct to three significant figures. [7]

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For Examiner's use only	
Question Number	**Marks**
1	
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