Rewarding Learning

ADVANCED
General Certificate of Education
2022

Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

[AMT11]

MONDAY 6 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

COVID-19 Context

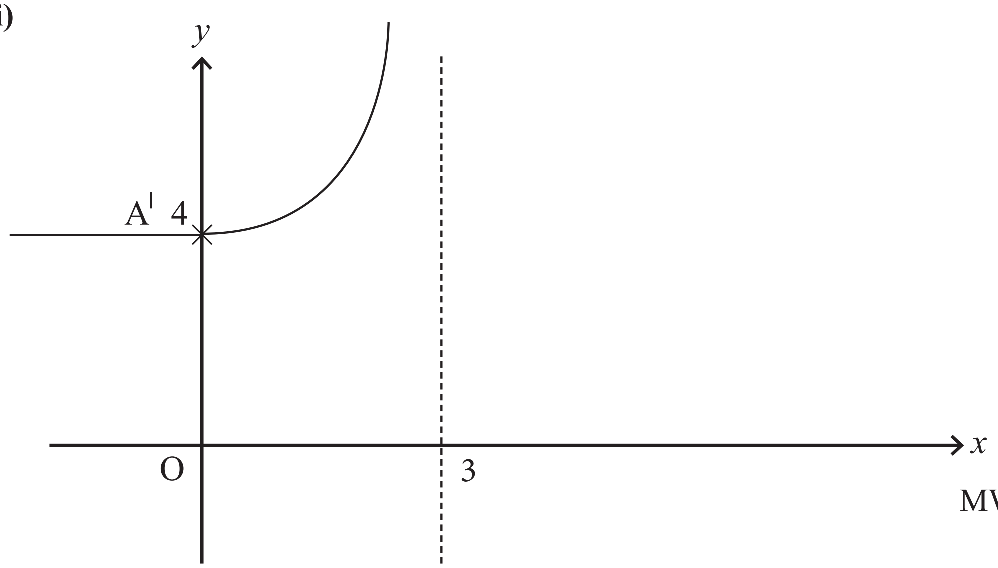
Given the unprecedented circumstances presented by the COVID-19 public health crisis, senior examiners, under the instruction of CCEA awarding organisation, are required to train assistant examiners to apply the mark scheme in case of disrupted learning and lost teaching time. The interpretation and intended application of the mark scheme for this examination series will be communicated through the standardising meeting by the Chief or Principal Examiner and will be monitored through the supervision period. This paragraph will apply to examination series in 2021–2022 only.

				AVAILABLE MARKS
1	**(i)**	$ar^3 = -40\frac{1}{2}$		
		$ar^4 = 30\frac{3}{8}$		
		$r = \frac{ar^4}{ar^3}$	M1 MW1	
		$r = \frac{30\frac{3}{8}}{-40\frac{1}{2}}$		
		$r = -\frac{3}{4}$	W1	
	(ii)	$a\left(-\frac{3}{4}\right)^3 = -40\frac{1}{2}$	M1	
		$a = 96$	W1	
	(iii)	$S_\infty = \frac{a}{1-r}$		
		$S_\infty = \frac{96}{1-\left(-\frac{3}{4}\right)}$	M1	
		$S_\infty = 54\frac{6}{7}$	MW1	7
2	**(i)**	$\frac{dy}{dx} = 3(2x+5)^4 + 3x[4(2x+5)^3](2)$	M1W1 MW1	
		$\frac{dy}{dx} = 3(2x+5)^4 + 24x(2x+5)^3$	W1	
	(ii)	$\frac{dy}{dx} = \frac{(5x^2+4)(1)-(x-7)(10x)}{(5x^2+4)^2}$	M1MW2	
		$\frac{dy}{dx} = \frac{-5x^2+70x+4}{(5x^2+4)^2}$	W1	
	(iii)	$\frac{dy}{dx} = 5(1+\ln 3x)^4\left(\frac{3}{3x}\right)$	M1 M1 W1	
		$\frac{dy}{dx} = \frac{5}{x}(1+\ln 3x)^4$	W1	12

		AVAILABLE MARKS
3 (a) (i) LHS		
$= \frac{1}{\cos 2\theta} \times \cos\theta$	M1W1	
$= \frac{\cos\theta}{2\cos^2\theta - 1}$	MW1	
Alternative solution		
RHS		
$= \frac{\cos\theta}{\cos 2\theta}$	M1W1	
$= \sec 2\theta \cos\theta$	MW1	
(ii) $\frac{\cos\theta}{2\cos^2\theta - 1} = 1$	M1	
$0 = 2\cos^2\theta - \cos\theta - 1$	MW1	
$(2\cos\theta + 1)(\cos\theta - 1) = 0$		
$\cos\theta = -\frac{1}{2}$ or $\cos\theta = 1$	MW1	
$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$ rad	MW2	
(b) (i) $\cos A\widehat{O}D = \frac{6^2 + 30^2 - (6\sqrt{21})^2}{2(6)(30)}$	M1W1	
$\cos A\widehat{O}D = \frac{1}{2}$		
$A\widehat{O}D = \frac{\pi}{3}$ rad	W1	
Angle at centre of sector DOC $D\widehat{O}C = \pi - 2A\widehat{O}D$ $= \frac{\pi}{3}$ rad	MW1	
(ii) Area of sector $= \frac{1}{2} r^2 D\widehat{O}C$		
$= \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$	M1	
$= 6\pi$ cm^2	W1	
Area of triangle $= \frac{1}{2} ab \sin D\widehat{O}C$		
$= \frac{1}{2}(6)(6)\sin\left(\frac{\pi}{3}\right)$	M1W1	
$= 18\sin\left(\frac{\pi}{3}\right)$		
$= 9\sqrt{3}$	W1	
Area of ice-cream $= (6\pi - 9\sqrt{3})$ cm^2	MW1	18

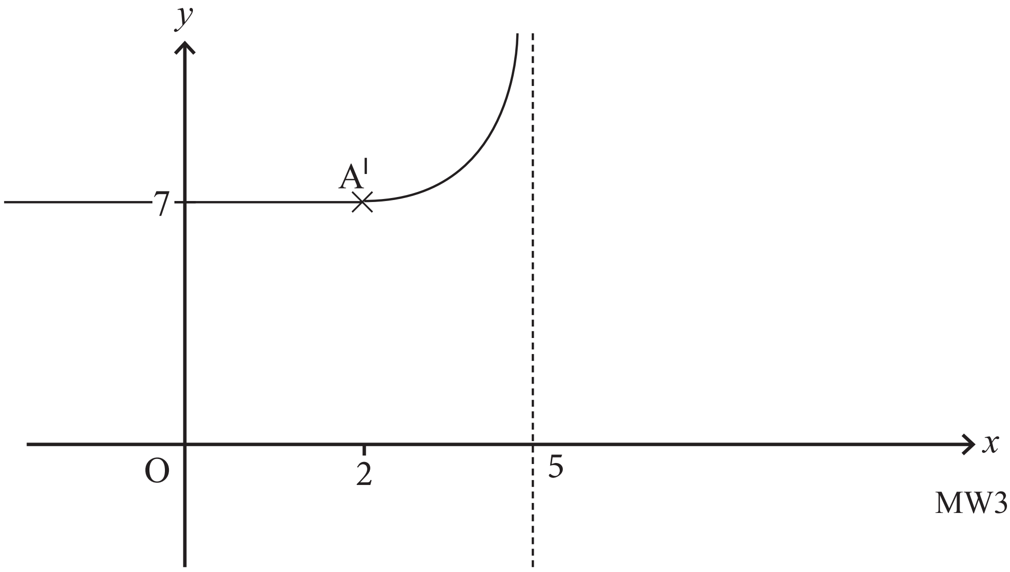
AVAILABLE MARKS

4 **(i)**



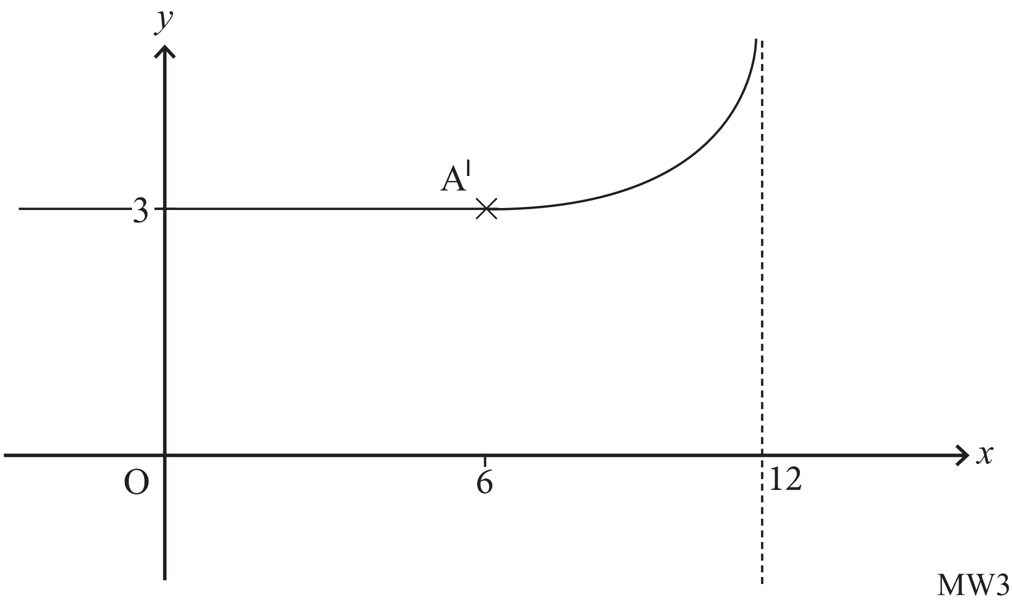
MW2

(ii)



MW3

(iii)



MW3

8

5 **(i)** $-[x^2 - 3x - 10]$ M1

$= -\left[\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 10\right]$ M1

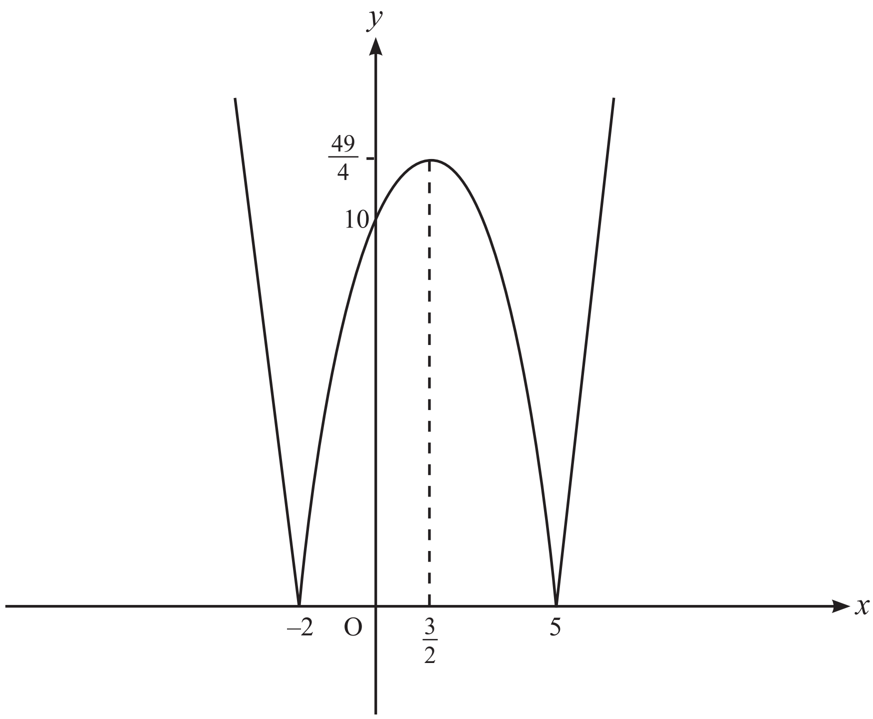
$= -\left(x - \frac{3}{2}\right)^2 + \frac{49}{4}$ W1

(ii) $f(x) \leqslant \frac{49}{4}$ MW1

(iii) $x > \frac{3}{2}$ MW1

(iv) $gf(x) = | -x^2 + 3x + 10|$ MW1

Cuts x-axis at $x = 5$, $x = -2$ MW1



MW3 10

AVAILABLE MARKS

			AVAILABLE MARKS
6	**(i)**	$\dfrac{x+5}{(3-x)(1+x)^2} \equiv \dfrac{A}{(3-x)} + \dfrac{B}{(1+x)} + \dfrac{C}{(1+x)^2}$	M1	

$x + 5 \equiv A(1+x)^2 + B(3-x)(1+x) + C(3-x)$ M1

When $x = 3$ M1

$8 = A(4)^2 + 0B + 0C$

$A = \dfrac{1}{2}$ W1

When $x = -1$

$4 = 0A + 0B + 4C$

$C = 1$ MW1

Equate x^2 terms

$0 = A - B$ M1

$B = \dfrac{1}{2}$ W1

$$\frac{x+5}{(3-x)(1+x)^2} \equiv \frac{1}{2(3-x)} + \frac{1}{2(1+x)} + \frac{1}{(1+x)^2}$$

(ii) $\dfrac{(x+5)}{(3-x)(1+x)^2} \equiv \dfrac{1}{2(3-x)} + \dfrac{1}{2(1+x)} + \dfrac{1}{(1+x)^2}$ M1

$\dfrac{1}{2(3-x)} = \dfrac{1}{2}(3-x)^{-1} = \dfrac{1}{2}\left(\dfrac{1}{3}\right)\left(1 - \dfrac{x}{3}\right)^{-1}$ MW1

$= \dfrac{1}{6}\left(1 + (-1)\left(-\dfrac{x}{3}\right) + \dfrac{(-1)(-2)}{2!}\left(-\dfrac{x}{3}\right)^2 + \ldots\right)$ M1

$= \dfrac{1}{6}\left(1 + \dfrac{x}{3} + \dfrac{x^2}{9} + \ldots\right)$ W1

$\dfrac{1}{2}(1+x)^{-1} = \dfrac{1}{2}\left(1 + (-1)x + \dfrac{(-1)(-2)}{2!}(x)^2 + \ldots\right)$

$= \dfrac{1}{2}(1 - x + x^2 + \ldots)$ M1W1

$(1+x)^{-2} = 1 + (-2)x + \dfrac{(-2)(-3)}{2!}(x)^2 + \ldots$

$= 1 - 2x + 3x^2 + \ldots$ M1W1

Hence expansion gives

$\dfrac{1}{6}\left(1 + \dfrac{x}{3} + \dfrac{x^2}{9} + \ldots\right) + \dfrac{1}{2}(1 - x + x^2 + \ldots) + (1 - 2x + 3x^2 + \ldots)$ M1

$= \dfrac{5}{3} - \dfrac{22}{9}x + \dfrac{95}{27}x^2$ W1 17

		AVAILABLE MARKS
7 **(i)** $\int \sin^2 x \, dx$		
$= \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$	M1W1	
$= \frac{x}{2} - \frac{1}{4} \sin 2x + c$	W3	
(ii) Attempt at using integration by parts	M1	
$\frac{dv}{dx} = e^{3x} \qquad u = x$	W1	
$v = \frac{1}{3} e^{3x} \qquad \frac{du}{dx} = 1$	MW2	
$\int x e^{3x} \, dx = \frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x} \, dx$	MW2	
$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + c$	W1	
(iii) $u^2 = x - 2$		
$2u \frac{du}{dx} = 1$		
$\frac{du}{dx} = \frac{1}{2u}$		
$dx = 2u du$	M1W1	
$x = u^2 + 2$	MW1	
$\int \frac{(u^2 + 2)^2}{u} 2u \, du$	M1W1	
$= \int 2(u^4 + 4u^2 + 4) \, du$		
$= \frac{2}{5} u^5 + \frac{8}{3} u^3 + 8u + c$	W1	
$\therefore \int \frac{x^2}{\sqrt{x - 2}} \, dx = \frac{2}{5} (\sqrt{x - 2})^5 + \frac{8}{3} (\sqrt{x - 2})^3 + 8\sqrt{x - 2} + c$	W1	19

			AVAILABLE MARKS
8	**(i)** $\frac{d\theta}{dt} = -k(\theta - 20)$		
	$\int \frac{d\theta}{\theta - 20} = \int -k \, dt$	M1W1	
	$\ln\lvert\theta - 20\rvert = -kt + c$	MW2	
	$\theta - 20 = Ae^{-kt}$	MW1	
	When $t = 0$, $\theta = 140$	M1	
	$A = 120$		
	$\theta = 120e^{-kt} + 20$	W1	
	(ii) $\theta = 120e^{-kt} + 20$		
	When $t = 2$, $\theta = 75$	M1	
	$k = 0.3901$	W1	
	$\theta = 120e^{-0.3901t} + 20$		
	$\therefore\ 120e^{-0.3901t} + 20 \geqslant 60$	M1	
	$\frac{40}{120} \leqslant e^{-0.3901t}$		
	$t \leqslant 2.816$ minutes	W1	
	2 minutes 49 seconds (nearest second)	MW1	12
9	$y(2x) + x^2 \frac{dy}{dx} + 4x = 10 \frac{dy}{dx}$	M1 MW3 W1	
	$\frac{dy}{dx} = \frac{-2xy - 4x}{x^2 - 10}$		
	When $x = \sqrt{2}$, $y = 0$		
	$\frac{dy}{dx} = \frac{\sqrt{2}}{2}$	MW1	
	Gradient of normal $= -\frac{2}{\sqrt{2}}$	MW1	
	$y = mx + c$, $(\sqrt{2}, 0)$	M1	
	$0 = -\frac{2}{\sqrt{2}}(\sqrt{2}) + c$		
	$c = 2$	MW1	
	$\therefore\ y = -\sqrt{2}\,x + 2$	MW1	10

		AVAILABLE MARKS
10 (i) $x = \cos\theta$		
$\frac{dx}{d\theta} = -\sin\theta$	MW1	
$y = \sin 2\theta$		
$\frac{dy}{d\theta} = 2\cos 2\theta$	MW1	
$\frac{dy}{dx} = 2\cos 2\theta \times -\frac{1}{\sin\theta}$	M1	
$\frac{dy}{dx} = -\frac{2\cos 2\theta}{\sin\theta}$	W1	
(ii) $0 = -\frac{2\cos 2\theta}{\sin\theta}$	M1	
$0 = -2\cos 2\theta$ and $\sin\theta \neq 0$	M1	
$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$	W2	
$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	W1	
(iii) When $\theta = \frac{\pi}{4}$		
$x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	M1	
$y = \sin\frac{2\pi}{4} = 1$	M1	
Repeated for all values of theta, or use symmetrical nature of the graph		
$\left(\frac{\sqrt{2}}{2}, 1\right)$, and $\left(\frac{-\sqrt{2}}{2}, -1\right)$	MW1	
$\left(\frac{-\sqrt{2}}{2}, 1\right)$, and $\left(\frac{\sqrt{2}}{2}, -1\right)$	W1	
(iv) $y = 2\sin\theta\cos\theta$	MW1	
$x^2 = \cos^2\theta,\ y^2 = 4\sin^2\theta\cos^2\theta$	MW1	
$y^2 = 4(1 - \cos^2\theta)\cos^2\theta$	MW1	
$y^2 = 4(1 - x^2)x^2$	W1	
(v) $V = \pi\int_a^b y^2\,dx$		
$V = \pi\int_0^{0.5} 4(1 - x^2)\,x^2\,dx$	M1W2	
$V = \pi\left[\frac{4}{3}x^3 - \frac{4}{5}x^5\right]_0^{0.5}$	MW2	
$V = \frac{17\pi}{120} = 0.445$ cubic units (3 sf)	W1	23

			AVAILABLE MARKS
11	**(i)** $g(x) = h(x)$		
	$\sqrt{3}\,x^{\frac{1}{2}} = \frac{1}{2}x^2 + x^5$	M1	
	$\sqrt{3}\,x^{\frac{1}{2}} - \frac{1}{2}x^2 - x^5 = 0$	MW1	
	Let $f(x) = \sqrt{3}\,x^{\frac{1}{2}} - \frac{1}{2}x^2 - x^5$		
	$f'(x) = \frac{\sqrt{3}}{2}x^{-\frac{1}{2}} - x - 5x^4$	W3	
	$x_1 \approx x_0 - \frac{f(x_0)}{f'(x_0)}$		
	$x_0 = 1$		
	$x_1 = 1 - \frac{\sqrt{3}(1) - \frac{1}{2}(1)^2 - 1^5}{\frac{\sqrt{3}}{2}(1)^{-\frac{1}{2}} - (1) - 5(1)^4}$	M1	
	$x_1 = 1.045$ (3 dp)	MW1	
	(ii) $\int_0^{1.045} \left(\sqrt{3}\,x^{\frac{1}{2}} - \frac{1}{2}x^2 - x^5\right) dx$	M2MW1	
	$= \left[\frac{2\sqrt{3}}{3}x^{\frac{3}{2}} - \frac{1}{6}x^3 - \frac{1}{6}x^6\right]_0^{1.045}$	MW3	
	$= 0.826$ sq units (3 sf)	W1	14
		Total	**150**