

For use in exams from the June 2017 Series onwards

## DATA - FUNDAMENTAL CONSTANTS AND VALUES

| Quantity   | Symbol          | Value                     | Units                             |
|--|-----------------|---------------------------|-----------------------------------|
| speed of light in vacuo                                      | $c$             | $3.00 \times 10^8$        | $\text{m s}^{-1}$                 |
| permeability of free space                                   | $\mu_0$         | $4\pi \times 10^{-7}$     | $\text{H m}^{-1}$                 |
| permittivity of free space                                   | $\epsilon_0$    | $8.85 \times 10^{-12}$    | $\text{F m}^{-1}$                 |
| magnitude of the charge of electron                          | $e$             | $1.60 \times 10^{-19}$    | C                                 |
| the Planck constant  | $h$             | $6.63 \times 10^{-34}$    | J s                               |
| gravitational constant                                       | $G$             | $6.67 \times 10^{-11}$    | $\text{N m}^2 \text{kg}^{-2}$     |
| the Avogadro constant  | $N_A$           | $6.02 \times 10^{23}$     | $\text{mol}^{-1}$                 |
| molar gas constant   | $R$             | 8.31                      | $\text{J K}^{-1} \text{mol}^{-1}$ |
| the Boltzmann constant                                       | $k$             | $1.38 \times 10^{-23}$    | $\text{J K}^{-1}$                 |
| the Stefan constant  | $\sigma$        | $5.67 \times 10^{-8}$     | $\text{W m}^{-2} \text{K}^{-4}$   |
| the Wien constant  | $\alpha$        | $2.90 \times 10^{-3}$     | m K                               |
| electron rest mass<br>(equivalent to $5.5 \times 10^{-4}$ u) | $m_e$           | $9.11 \times 10^{-31}$    | kg                                |
| magnitude of electron charge/mass ratio                      | $\frac{e}{m_e}$ | $1.76 \times 10^{11}$     | $\text{C kg}^{-1}$                |
| proton rest mass<br>(equivalent to 1.00728 u)                | $m_p$           | $1.67(3) \times 10^{-27}$ | kg                                |
| proton charge/mass ratio                                     | $\frac{e}{m_p}$ | $9.58 \times 10^7$        | $\text{C kg}^{-1}$                |
| neutron rest mass<br>(equivalent to 1.00867 u)               | $m_n$           | $1.67(5) \times 10^{-27}$ | kg                                |
| gravitational field strength                                 | $g$             | 9.81                      | $\text{N kg}^{-1}$                |
| acceleration due to gravity                                  | $g$             | 9.81                      | $\text{m s}^{-2}$                 |
| atomic mass unit<br>(1u is equivalent to 931.5 MeV)          | u               | $1.661 \times 10^{-27}$   | kg                                |

### ALGEBRAIC EQUATION

quadratic equation  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### ASTRONOMICAL DATA

| Body  | Mass/kg               | Mean radius/m      |
|-------|-----------------------|--------------------|
| Sun   | $1.99 \times 10^{30}$ | $6.96 \times 10^8$ |
| Earth | $5.97 \times 10^{24}$ | $6.37 \times 10^6$ |

### GEOMETRICAL EQUATIONS

arc length =  $r\theta$   
 circumference of circle =  $2\pi r$   
 area of circle =  $\pi r^2$   
 curved surface area of cylinder =  $2\pi r h$   
 surface area of sphere =  $4\pi r^2$   
 volume of sphere =  $\frac{4}{3}\pi r^3$

## Particle Physics

| Class   | Name        | Symbol    | Rest energy/MeV |
|---------|-------------|-----------|-----------------|
| photon  | photon      | $\gamma$  | 0               |
| lepton  | neutrino    | $\nu_e$   | 0               |
|         |             | $\nu_\mu$ | 0               |
|         | electron    | $e^\pm$   | 0.510999        |
|         | muon        | $\mu^\pm$ | 105.659         |
| mesons  | $\pi$ meson | $\pi^\pm$ | 139.576         |
|         |             | $\pi^0$   | 134.972         |
|         | K meson     | $K^\pm$   | 493.821         |
|         |             | $K^0$     | 497.762         |
| baryons | proton      | p         | 938.257         |
|         | neutron     | n         | 939.551         |

## Properties of quarks

antiquarks have opposite signs

| Type     | Charge          | Baryon number  | Strangeness |
|----------|-----------------|----------------|-------------|
| <b>u</b> | $+\frac{2}{3}e$ | $+\frac{1}{3}$ | 0           |
| <b>d</b> | $-\frac{1}{3}e$ | $+\frac{1}{3}$ | 0           |
| <b>s</b> | $-\frac{1}{3}e$ | $+\frac{1}{3}$ | -1          |

## Properties of Leptons

|                |  | Lepton number |
|----------------|--|---------------|
| Particles:     | $e^-, \nu_e; \mu^-, \nu_\mu$             | +1            |
| Antiparticles: | $e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$ | -1            |

## Photons and energy levels

photon energy  $E = hf = \frac{hc}{\lambda}$

photoelectricity  $hf = \phi + E_{k(\max)}$

energy levels  $hf = E_1 - E_2$

de Broglie wavelength  $\lambda = \frac{h}{p} = \frac{h}{mv}$

## Waves

wave speed  $c = f\lambda$     period  $f = \frac{1}{T}$

first harmonic  $f = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$

fringe spacing  $w = \frac{\lambda D}{s}$     diffraction grating  $d \sin \theta = n\lambda$

refractive index of a substance s,  $n = \frac{c}{c_s}$

for two different substances of refractive indices  $n_1$  and  $n_2$ ,  
law of refraction  $n_1 \sin \theta_1 = n_2 \sin \theta_2$

critical angle  $\sin \theta_c = \frac{n_2}{n_1}$  for  $n_1 > n_2$

## Mechanics

moments    moment =  $Fd$

velocity and acceleration  $v = \frac{\Delta s}{\Delta t}$      $a = \frac{\Delta v}{\Delta t}$

equations of motion  $v = u + at$      $s = \left(\frac{u+v}{2}\right)t$

$v^2 = u^2 + 2as$      $s = ut + \frac{at^2}{2}$

force  $F = ma$

force  $F = \frac{\Delta(mv)}{\Delta t}$

impulse  $F \Delta t = \Delta(mv)$

work, energy and power  $W = F s \cos \theta$

$E_k = \frac{1}{2} m v^2$      $\Delta E_p = mg\Delta h$

$P = \frac{\Delta W}{\Delta t}, P = Fv$

efficiency =  $\frac{\text{useful output power}}{\text{input power}}$

## Materials

density  $\rho = \frac{m}{V}$     Hooke's law  $F = k \Delta L$

Young modulus =  $\frac{\text{tensile stress}}{\text{tensile strain}}$     tensile stress =  $\frac{F}{A}$

tensile strain =  $\frac{\Delta L}{L}$

energy stored  $E = \frac{1}{2} F \Delta L$

**Electricity**

*current and pd*  $I = \frac{\Delta Q}{\Delta t}$   $V = \frac{W}{Q}$   $R = \frac{V}{I}$

*resistivity*  $\rho = \frac{RA}{L}$

*resistors in series*  $R_T = R_1 + R_2 + R_3 + \dots$

*resistors in parallel*  $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

*power*  $P = VI = I^2R = \frac{V^2}{R}$

*emf*  $\varepsilon = \frac{E}{Q}$   $\varepsilon = I(R + r)$

**Circular motion**

*magnitude of angular speed*  $\omega = \frac{v}{r}$

$\omega = 2\pi f$

*centripetal acceleration*  $a = \frac{v^2}{r} = \omega^2 r$

*centripetal force*  $F = \frac{mv^2}{r} = m\omega^2 r$

**Simple harmonic motion**

*acceleration*  $a = -\omega^2 x$

*displacement*  $x = A \cos(\omega t)$

*speed*  $v = \pm \omega \sqrt{(A^2 - x^2)}$

*maximum speed*  $v_{\max} = \omega A$

*maximum acceleration*  $a_{\max} = \omega^2 A$

*for a mass-spring system*  $T = 2\pi \sqrt{\frac{m}{k}}$

*for a simple pendulum*  $T = 2\pi \sqrt{\frac{l}{g}}$

**Thermal physics**

*energy to change temperature*  $Q = mc\Delta\theta$

*energy to change state*  $Q = ml$

*gas law*  $pV = nRT$   
 $pV = NkT$

*kinetic theory model*  $pV = \frac{1}{3}Nm(c_{\text{rms}})^2$

*kinetic energy of gas molecule*  $\frac{1}{2}m(c_{\text{rms}})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$

**Gravitational fields**

*force between two masses*  $F = \frac{Gm_1m_2}{r^2}$

*gravitational field strength*  $g = \frac{F}{m}$

*magnitude of gravitational field strength in a radial field*  $g = \frac{GM}{r^2}$

*work done*  $\Delta W = m\Delta V$

*gravitational potential*  $V = -\frac{GM}{r}$

$g = -\frac{\Delta V}{\Delta r}$

**Electric fields and capacitors**

*force between two point charges*  $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1Q_2}{r^2}$

*force on a charge*  $F = EQ$

*field strength for a uniform field*  $E = \frac{V}{d}$

*work done*  $\Delta W = Q\Delta V$

*field strength for a radial field*  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

*electric potential*  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

*field strength*  $E = \frac{\Delta V}{\Delta r}$

*capacitance*  $C = \frac{Q}{V}$

$C = \frac{A\epsilon_0\epsilon_r}{d}$

*capacitor energy stored*  $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2} \frac{Q^2}{C}$

*capacitor charging*  $Q = Q_0(1 - e^{-\frac{t}{RC}})$

*decay of charge*  $Q = Q_0e^{-\frac{t}{RC}}$

*time constant*  $RC$

## Magnetic fields

|                                       |   |
|---------------------------------------|---|
| <i>force on a current</i>             | $F = BIl$   |
| <i>force on a moving charge</i>       | $F = BQv$   |
| <i>magnetic flux</i>                  | $\Phi = BA$   |
| <i>magnetic flux linkage</i>          | $N\Phi = BAN \cos \theta$   |
| <i>magnitude of induced emf</i>       | $\varepsilon = N \frac{\Delta\Phi}{\Delta t}$                                       |
|                                       | $N\Phi = BAN \cos \theta$   |
| <i>emf induced in a rotating coil</i> | $\varepsilon = BAN\omega \sin \omega t$   |
| <i>alternating current</i>            | $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \quad V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ |
| <i>transformer equations</i>          | $\frac{N_s}{N_p} = \frac{V_s}{V_p}$   |
|                                       | $\text{efficiency} = \frac{I_s V_s}{I_p V_p}$                                       |

## Nuclear physics

|   |  |
|---|--|
| <i>inverse square law for <math>\gamma</math> radiation</i> | $I = \frac{k}{x^2}$  |
| <i>radioactive decay</i>                                    | $\frac{\Delta N}{\Delta t} = -\lambda N, N = N_0 e^{-\lambda t}$ |
| <i>activity</i>   | $A = \lambda N$  |
| <i>half-life</i>  | $T_{1/2} = \frac{\ln 2}{\lambda}$                                |
| <i>nuclear radius</i>                                       | $R = R_0 A^{1/3}$  |
| <i>energy-mass equation</i>                                 | $E = mc^2$   |

## OPTIONS

### Astrophysics

|  |  |
|--|--|
| 1 astronomical unit  | $= 1.50 \times 10^{11} \text{ m}$  |
| 1 light year   | $= 9.46 \times 10^{15} \text{ m}$  |
| 1 parsec   | $= 2.06 \times 10^5 \text{ AU} = 3.08 \times 10^{16} \text{ m}$<br>$= 3.26 \text{ ly}$ |
| Hubble constant, $H$   | $= 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$  |
| $M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$ |  |
| <i>telescope in normal adjustment</i>  | $M = \frac{f_o}{f_e}$  |
| <i>Rayleigh criterion</i>  | $\theta \approx \frac{\lambda}{D}$   |
| <i>magnitude equation</i>  | $m - M = 5 \log \frac{d}{10}$  |
| <i>Wien's law</i>  | $\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K}$                              |
| <i>Stefan's law</i>  | $P = \sigma AT^4$  |
| <i>Schwarzschild radius</i>  | $R_s \approx \frac{2GM}{c^2}$  |
| <i>Doppler shift for <math>v \ll c</math></i>  | $\frac{\Delta f}{f} = -\frac{\Delta \lambda}{\lambda} = \frac{v}{c}$                   |
| <i>red shift</i>   | $z = -\frac{v}{c}$   |
| <i>Hubble's law</i>  | $v = Hd$   |

### Medical physics

|                             |   |
|-----------------------------|---|
| <i>lens equations</i>       | $P = \frac{1}{f}$<br>$m = \frac{v}{u}$<br>$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ |
| <i>threshold of hearing</i> | $I_0 = 1.0 \times 10^{-12} \text{ W m}^{-2}$  |
| <i>intensity level</i>      | $\text{intensity level} = 10 \log \frac{I}{I_0}$                                    |
| <i>absorption</i>           | $I = I_0 e^{-\mu x}$<br>$\mu_m = \frac{\mu}{\rho}$                                  |
| <i>ultrasound imaging</i>   | $Z = p c$<br>$\frac{I_r}{I_i} = \left( \frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2$       |
| <i>half-lives</i>           | $\frac{1}{T_E} = \frac{1}{T_B} + \frac{1}{T_P}$                                     |

Engineering physics

|                             |  |
|-----------------------------|--|
| moment of inertia           | $I = \Sigma mr^2$  |
| angular kinetic energy      | $E_k = \frac{1}{2} I \omega^2$   |
| equations of angular motion | $\omega_2 = \omega_1 + \alpha t$<br>$\omega_2^2 = \omega_1^2 + 2\alpha\theta$<br>$\theta = \omega_1 t + \frac{\alpha t^2}{2}$<br>$\theta = \frac{(\omega_1 + \omega_2) t}{2}$  |
| torque                      | $T = I \alpha$<br>$T = F r$  |
| angular momentum            | angular momentum = $I\omega$   |
| angular impulse             | $T\Delta t = \Delta(I\omega)$  |
| work done                   | $W = T\theta$  |
| power                       | $P = T\omega$  |
| thermodynamics              | $Q = \Delta U + W$<br>$W = p\Delta V$  |
| adiabatic change            | $pV^\gamma = \text{constant}$  |
| isothermal change           | $pV = \text{constant}$   |
| heat engines                | efficiency = $\frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$<br>maximum theoretical efficiency = $\frac{T_H - T_C}{T_H}$<br>work done per cycle = area of loop<br>input power = calorific value $\times$ fuel flow rate<br>indicated power = (area of $p - V$ loop) $\times$ (number of cycles per second) $\times$ (number of cylinders)<br>output or brake power $P = T\omega$<br>friction power = indicated power - brake power<br>heat pumps and refrigerators<br>refrigerator: $COP_{\text{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$<br>heat pump: $COP_{\text{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C}$ |

Turning points in physics

|                       |   |
|-----------------------|---|
| electrons in fields   | $F = \frac{eV}{d}$<br>$F = Bev$<br>$r = \frac{mv}{Be}$<br>$\frac{1}{2} mv^2 = eV$   |
| Millikan's experiment | $\frac{QV}{d} = mg$<br>$F = 6\pi\eta r v$   |
| Maxwell's formula     | $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$<br>$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$  |
| special relativity    | $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$<br>$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$<br>$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$ |

Electronics

|                                   |  |
|-----------------------------------|--|
| resonant frequency                | $f_0 = \frac{1}{2\pi \sqrt{LC}}$   |
| Q-factor                          | $Q = \frac{f_0}{f_B}$  |
| operational amplifiers: open loop | $V_{\text{out}} = A_{OL}(V_+ - V_-)$   |
| inverting amplifier               | $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$                                |
| non-inverting amplifier           | $\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$                                       |
| summing amplifier                 | $V_{\text{out}} = -R_f \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots \right)$ |
| difference amplifier              | $V_{\text{out}} = (V_+ - V_-) \frac{R_f}{R_1}$   |
| Bandwidth requirement:            |  |
| for AM                            | bandwidth = $2f_M$   |
| for FM                            | bandwidth = $2(\Delta f + f_M)$  |





