AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 3 Statistics

Thursday 11 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 2 hours to complete **both** papers.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 50.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
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5	
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7	
8	
9	
TOTAL	

Answer **all** questions in the spaces provided.

1 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{5} & 1 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X \ge 3)$

Circle your answer.

[1 mark]

$\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$

2 Jamie is conducting a hypothesis test on a random variable which has a normal distribution with standard deviation 1

The hypotheses are

$H_0 : \mu = 5$

$H_1 : \mu > 5$

He takes a random sample of size 4

The mean of his sample is 6

He uses a 5% level of significance.

Before Jamie conducted the test, what was the probability that he would make a Type I error?

Circle your answer.

[1 mark]

0.0228 0.0456 0.0500 0.1587

3 The mass of male giraffes is assumed to have a normal distribution.

Duncan takes a random sample of 600 male giraffes.

The mean mass of the sample is 1196 kilograms.

The standard deviation of the sample is 98 kilograms.

3 (a) Construct a 94% confidence interval for the mean mass of male giraffes, giving your values to one decimal place.

[3 marks]

3 (b) Explain whether or not your answer to part **(a)** would change if a sample of size 5 was taken with the same mean and standard deviation.

[1 mark]

Turn over for the next question

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4 The discrete random variable X follows a discrete uniform distribution and takes values 1, 2, 3, …, n.

The discrete random variable Y is defined by $Y = 2X$

4 (a) Using the standard results for $\sum n$, $\sum n^2$ and $\text{Var}(aX + b)$, prove that

$$\text{Var}(Y) = \frac{n^2 - 1}{3}$$

[7 marks]

Do not write outside the box

4 (b) A spinning toy can land on one of four values: 2, 4, 6 or 8

Using a discrete uniform distribution, find the probability that the next value the toy lands on is greater than 2

[1 mark]

4 (c) State an assumption that is required for the discrete uniform distribution used in part **(b)** to be valid.

[1 mark]

Turn over for the next question

Do not write outside the box

5 Emily claims that the average number of runners per minute passing a shop during a long distance run is 8

Emily conducts a hypothesis test to investigate her claim.

5 (a) State the hypotheses for Emily's test.

[1 mark]

5 (b) Emily counts the number of runners, X, passing the shop in a randomly chosen minute.

The critical region for Emily's test is $X \leq 2$ or $X \geq 14$

During a randomly chosen minute, Emily counts 3 runners passing the shop.

Determine the outcome of Emily's hypothesis test.

[3 marks]

Do not write outside the box

5 (c) The actual average number of runners per minute passing the shop is 7

Find the power of Emily's hypothesis test, giving your answer to three significant figures.

[3 marks]

Turn over for the next question

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6 The distance, X metres, between successive breaks in a water pipe is modelled by an exponential distribution. The mean of X is 25

The distance between two successive breaks is measured. A water pipe is given a 'Red' rating if the distance is less than d metres.

The government has introduced a new law changing d to 2

Before the government introduced the new law, the probability that a water pipe is given a 'Red' rating was 0.05

6 (a) Explain whether or not the probability that a water pipe is given a 'Red' rating has increased as a result of the new law.

[4 marks]

Do not write outside the box

6 (b) Find the probability density function of the random variable X.

[2 marks]

6 (c) After investigation, the distances between successive breaks in water pipes are found to have a standard deviation of 5 metres.

Explain whether or not the use of an exponential model in parts **(a)** and **(b)** is appropriate.

[2 marks]

Turn over for the next question

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7 The rainfall per day in February in a particular town has been recorded as having a mean of 1.8 inches.

Sienna claims that rainfall in February has increased in the town. She records the rainfall in a random sample of 12 days.

Her sample mean is 2 inches and her sample standard deviation is 0.4 inches.

It is assumed that rainfall per day has a normal distribution.

7 (a) Investigate Sienna's claim using the 5% level of significance.

[6 marks]

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7 (b) For the test carried out in part **(a)**, state in context the meaning of a Type II error.
[1 mark]

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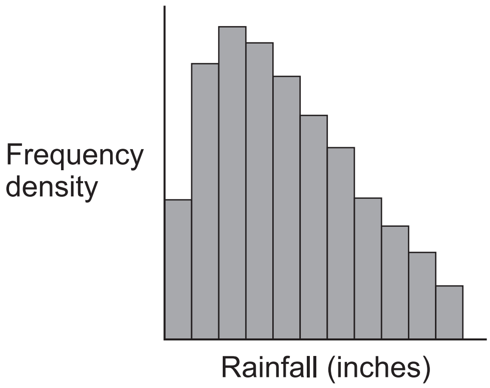
7 (c) The distribution of rainfall per day in February in the town over 10 years is shown in the histogram.



Explain whether or not the assumption that rainfall per day in February has a normal distribution is appropriate.
[1 mark]

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Turn over for the next question

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8 Ray is conducting a hypothesis test with the hypotheses

H_0 : There is no association between time of day and number of snacks eaten

H_1 : There is an association between time of day and number of snacks eaten

He calculates **expected** frequencies correct to two decimal places, which are given in the following table.

		Number of snacks eaten		
		0	1	2 or more
Time of Day	Day	23.68	21.05	5.26
	Night	21.32	18.95	4.74

Ray calculates his test statistic using $\sum \frac{(O-E)^2}{E}$

8 (a) State, with a reason, the error Ray has made and describe any changes Ray will need to make to his test.

[3 marks]

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8 (b) Having made the necessary corrections as described in part **(a)**, the correct value of the test statistic is 8.74

Complete Ray's hypothesis test using a 1% level of significance.

[3 marks]

Turn over for the next question

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9 The continuous random variable X has the cumulative distribution function shown below.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{62}(4x^3 + 6x^2 + 3x) & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

The discrete random variable Y has the probability distribution shown below.

y	2	7	13	19
$P(Y = y)$	0.5	0.1	0.1	0.3

The random variables X and Y are independent.

Find the exact value of $E(X^3 + Y)$.

[6 marks]

END OF QUESTIONS

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