A-level
FURTHER MATHEMATICS
7367/3M

Paper 3 Mechanics

Mark scheme

June 2023

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2 3 6 A 7 3 6 7 / 3 M / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.2	B1	ML^2T^{-3}
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	1.1b	B1	6 N m
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Circles correct answer	1.1b	B1	6.4 cm
	Question total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Obtains a correct value or expression for the acceleration.	1.1a	M1	$F = 1400 \times \frac{12^2}{60}$ $= 3360$ N
	Obtains the correct friction. Accept 3400. Condone missing units.	1.1b	A1	
	Question total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Deduces correct limits for the integration. Can be seen at any point in their working.	2.2a	B1	$\text{Mass} = \rho\pi\int_0^2 \left(4-x^2\right)^2 dx = \frac{256}{15}\pi\rho$ $\frac{256}{15}\pi\rho y\bar{x} = \rho\pi\int_0^2 x\left(4-x^2\right)^2 dx$ $\frac{256}{15}\bar{x} = \frac{32}{3}$ $\bar{x} = \frac{480}{768} = \frac{5}{8}$
	Obtains correct value or expression for the mass or volume. PI by seeing $\frac{256}{15}$	3.3	B1	
	Sets up an expression containing an integral to find centre of mass. Condone $\int xy\,\mathrm{d}x$ Condone missing ρ	1.1a	M1	
	Evaluates the integral on RHS correctly. PI by seeing $\frac{32}{3}$	1.1b	A1	
	Completes a reasoned argument that includes the density and π to obtain the required result.	2.1	R1	
	Subtotal		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	Uses a trigonometric function to form an equation to find the angle. Must include two of 4, $\frac{5}{8}$ and AWRT 4.05	3.4	M1	$\tan\theta = \frac{32}{5}$ $\theta = 81°$
	Obtains the correct angle.	1.1b	A1	
	Subtotal		**2**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Forms an energy equation with GPE and KE terms.	3.3	M1	$0.8\times10\times2(1-\cos30°)=\frac{1}{2}\times0.8\times v^2$ $v=\sqrt{40(1-\cos30°)}$ $v=2.31\ =2\text{ m s}^{-1}$ (to 1 sf)
	Obtains correct equation.	1.1b	A1	
	Obtains correct speed giving their answer to 1 sf. Condone missing units.	3.2a	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)	States that the string is inextensible or inelastic or that the string is light.	1.1b	B1	String is inextensible
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(c)	Forms an energy equation involving $\cos\theta$	3.3	M1	$mgr(\cos\theta-\cos30°)=\frac{1}{2}mv^2$ $v^2=2gr(\cos\theta-\cos30°)$ $v^2=40\cos\theta-20\sqrt{3}$ $T-mg\cos\theta=\frac{mv^2}{r}$ $T-mg\cos\theta=2mg(\cos\theta-\cos30°)$ $T=3mg\cos\theta-2mg\cos30°$ $T=24\cos\theta-16\cos30°$ $T=24\cos\theta-8\sqrt{3}$ For maximum T require $\cos\theta=1$ so that $\theta=0$ which is when the particle is directly below O.
	Forms a three term equation of motion by resolving radially.	3.3	M1	
	Resolves radially to obtain a correct equation in terms of T, v and θ	1.1b	A1	
	Combines their equations to eliminate v.	3.4	M1	
	Deduces angle for maximum tension from their equation.	2.2a	B1	
	Completes a reasoned argument to obtain the required result.	2.1	R1	
	Subtotal		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(d)	Evaluates maximum tension. Follow through their expression for the tension.	1.1b	B1F	$T = 24 - 8\sqrt{3} = 10.14... \text{ N}$ Air resistance would reduce the speed of the sphere slightly and hence tension when the sphere is directly below *O*, so the result obtained is reasonable as it is a little less than the predicted tension.
	Explains that air resistance would reduce speed and hence tension when the string is vertical, so the result obtained is reasonable as it is less than the predicted tension. Or explains that g will be less than 10 and that this would result in a lower value for the tension. This mark is only awarded if a value greater than 9.5 is obtained and explained in a reasonable way	3.5a	E1	
	Subtotal		**2**	

	Question total		**12**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Writes the velocities of *A* and *B* as vectors or in components.	3.3	M1	$\mathbf{u}_A = \begin{pmatrix} 4\cos 30° \\ 4\sin 30° \end{pmatrix}$
	Both velocities or all components correct.	1.1b	A1	$\mathbf{u}_B = \begin{pmatrix} 6\cos 45° \\ -6\sin 45° \end{pmatrix}$
	Forms an equation using conservation of momentum with vectors or forms two equations if working with components.	3.4	M1	$2\begin{pmatrix} 4\cos 30° \\ 4\sin 30° \end{pmatrix} + 3\begin{pmatrix} 6\cos 45° \\ -6\sin 45° \end{pmatrix} = 5\mathbf{v}$ $\mathbf{v} = \frac{1}{5}\begin{pmatrix} 8\cos 30° + 18\cos 45° \\ 8\sin 30° - 18\sin 45° \end{pmatrix}$
	Obtains a correct equation(s).	1.1b	A1	$\mathbf{I}_A = \frac{2}{5}\begin{pmatrix} 8\cos 30° + 18\cos 45° \\ 8\sin 30° - 18\sin 45° \end{pmatrix} - 2\begin{pmatrix} 4\cos 30° \\ 4\sin 30° \end{pmatrix}$ $= \begin{pmatrix} 0.934 \\ -7.491 \end{pmatrix}$
	Obtains correct value for velocity, either in vector form or as components.	1.1b	A1	$\lvert\mathbf{I}_A\rvert = \sqrt{0.934^2 + 7.491^2}$ $= 7.549...$ $= 7.55$ N s (to 3 sf)
	Recalls required equation for impulse	1.2	B1	
	Uses their velocities in the impulse equation with vectors or forms two equations if working with components.	3.4	M1	
	Obtains the correct magnitude of impulse from a reasoned argument.	2.1	R1	
	Subtotal		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(b)	States 7.55 N s Must be positive	1.1b	B1	7.55 N s because the impulse on *B* is equal in size but opposite in *direction* to *A*.
	Explains that the impulse on *B* is equal in size but opposite in direction to *A*.	2.4	E1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(c)	Uses a trigonometric function to form an equation to find the angle. Must include two of AWRT 7.55, AWRT 0.93 and AWRT 7.5 or the corresponding values from their impulse vector in part (a).	3.4	M1	$\tan\alpha = \frac{7.491}{0.934}$ $\tan\alpha = 8.018$ $\alpha = 83°$
	Obtains correct angle. Allow AWRT ±83°. Accept 97°	1.1b	A1	Angle is 83°
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(d)	States that *B* crosses the line and gives a valid reason based on the sign of the velocity component perpendicular to the line.	2.4	E1	Particle *B* crosses the line as the sign of its velocity component perpendicular to the line does not change.
	Subtotal		**1**	

	Question total		**13**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Obtains the correct normal reaction.	3.3	B1	$R = 10 \times 9.8 - 100\sin 30° = 48$ Let d be the distance moved by the block. $100\cos 30° = 0.6 \times 48 + \frac{40}{2}(d-1)$ $d = 3.890$ $= 3.9$ m (to 2sf)
	Uses Hooke's law to form a correct expression for the tension in terms of a variable.	3.3	B1	
	Forms a three term equation for equilibrium with at least two terms correct but allowing their R.	1.1a	M1	
	Completes a reasoned solution to obtain the required value.	2.1	R1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	States that air resistance has been ignored.	3.5b	B1	Air resistance has been ignored.
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(c)	Identifies that the maximum speed is when the forces are in equilibrium. PI By seeing 3.89 or 2.89	3.4	B1	$100\cos 30° \times 3.89 - 0.6 \times 48 \times 3.89 - \frac{1}{2} \times \frac{40}{2} \times 2.89^2$ $= \frac{1}{2} \times 10\, v^2$ $v = 5.317 = 5.3$ m s^{-1} (to 2 sf)
	Obtains at least three of the four work/energy terms with at least two terms correct (either expressions or values).	3.3	M1	
	Obtains correct energy equation.	1.1b	A1	
	Obtains correct speed.	1.1b	A1	
	Subtotal		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(d)	Forms an energy-based argument, with at least two correct energy expressions	3.3	M1	$100\cos 30° x - 0.6 \times 48 \times x - \frac{1}{2} \times \frac{40}{2} \times (x-1)^2 = 0$ $-10x^2 + (100\cos 30° - 28.8 + 20)x - 10 = 0$ $10x^2 - 77.80x + 10 = 0$ $x = 7.65$ Distance from $O = 8.65$ m (to 3sf) Block may reach the wall if the length of block is great enough to account for the 5 cm shortfall.
	Obtains exactly three correct energy expressions.	1.1b	A1	
	Obtains a correct conclusion to their three term energy argument. PI by correct distance from *O*	1.1b	A1F	
	Writes coherent reason to justify their answer with reference to the size of the block.	2.2b	E1	
	Subtotal		**4**	

	Question total		**13**	

	Paper total		**50**	