AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 3

Friday 12 June 2020 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Given that

$$\int_0^{10} \mathrm{f}(x)\,\mathrm{d}x = 7$$

deduce the value of

$$\int_0^{10} \Big(\mathrm{f}(x) + 1\Big)\,\mathrm{d}x$$

Circle your answer.

[1 mark]

-3 7 8 17

2 Given that

$$6\cos\theta + 8\sin\theta \equiv R\cos(\theta + \alpha)$$

find the value of R.

Circle your answer.

[1 mark]

6 8 10 14

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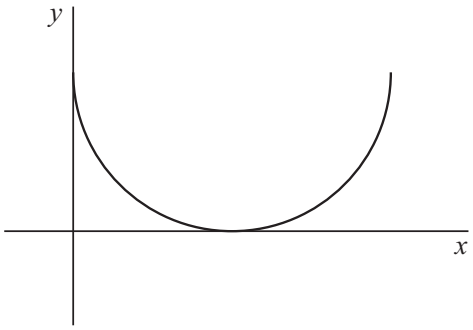
3 Determine which one of these graphs does **not** represent y as a function of x.

Tick (✓) **one** box.

[1 mark]



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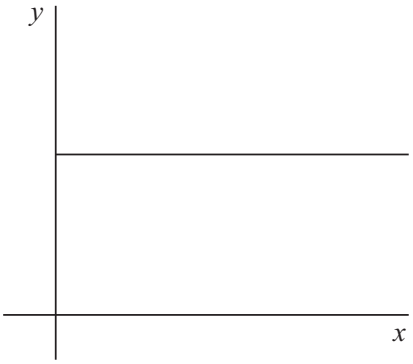

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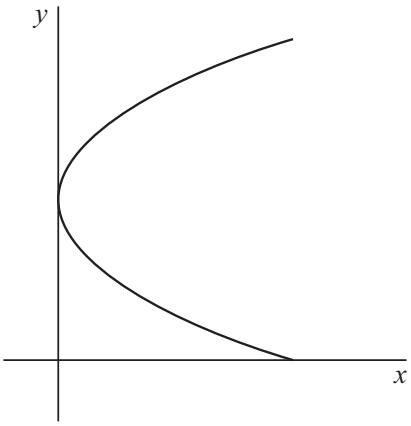

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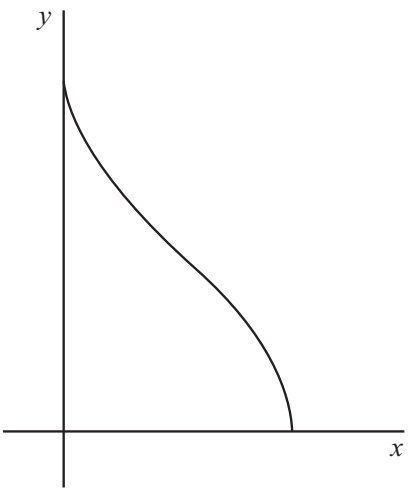

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4 $\mathrm{p}(x) = 4x^3 - 15x^2 - 48x - 36$

4 (a) Use the factor theorem to prove that $x - 6$ is a factor of $\mathrm{p}(x)$.

[2 marks]

4 (b) (i) Prove that the graph of $y = \mathrm{p}(x)$ intersects the x-axis at exactly one point.

[4 marks]

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4 (b) (ii) State the coordinates of this point of intersection.

[1 mark]

Turn over for the next question

5 The number of radioactive atoms, N, in a sample of a sodium isotope after time t hours can be modelled by

$$N = N_0 e^{-kt}$$

where N_0 is the initial number of radioactive atoms in the sample and k is a positive constant.

The model remains valid for large numbers of atoms.

5 (a) It takes 15.9 hours for half of the sodium atoms to decay.

Determine the number of days required for at least 90% of the number of atoms in the original sample to decay.

[5 marks]

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5 (b) Find the percentage of the atoms remaining after the first week.

Give your answer to two significant figures.

[2 marks]

5 (c) Explain why the model can only provide an estimate for the number of remaining atoms.

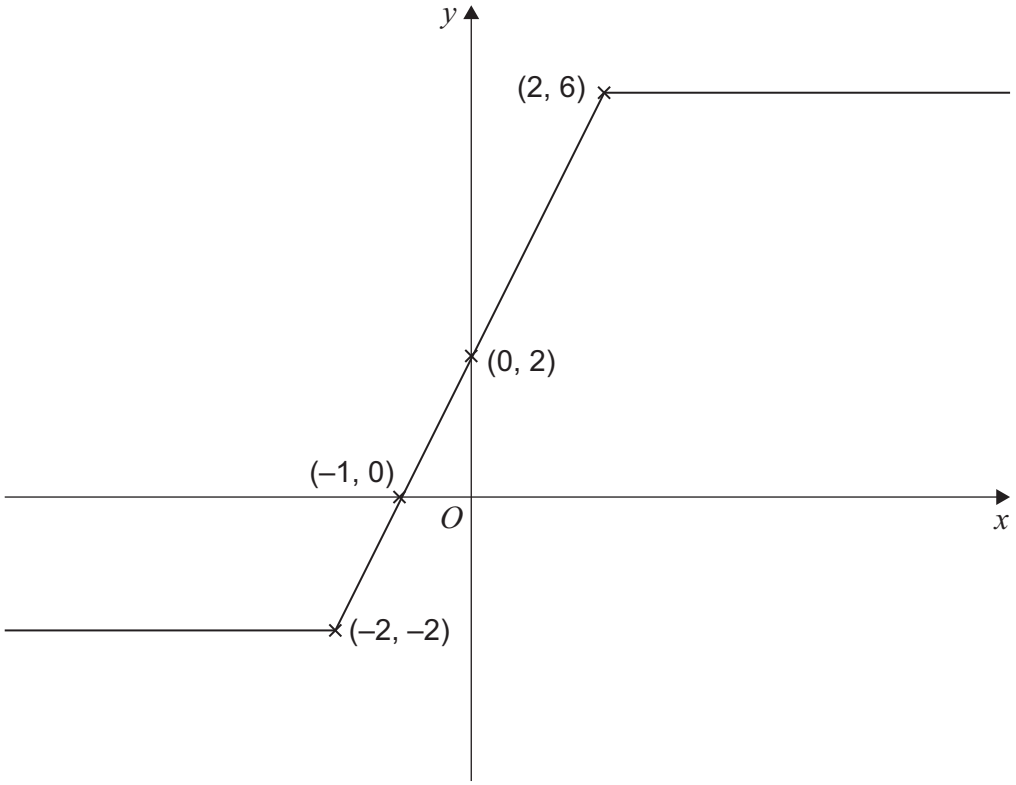
[1 mark]

5 (d) Explain why the model is invalid in the long run.

[1 mark]

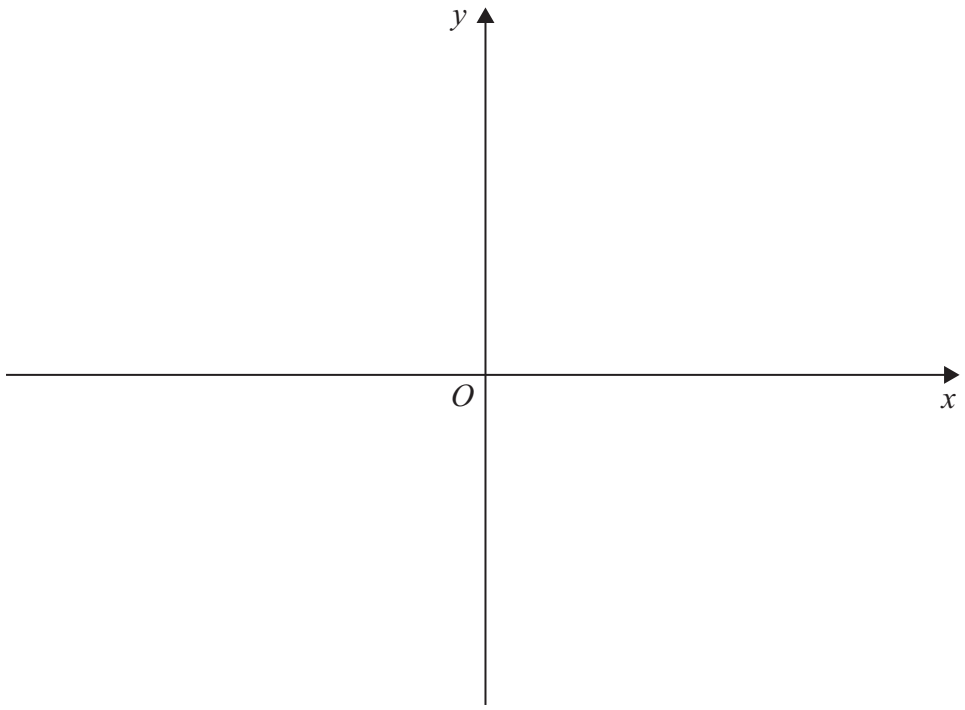
Turn over for the next question

6 The graph of $y = f(x)$ is shown below.



6 (a) Sketch the graph of $y = f(-x)$

[2 marks]



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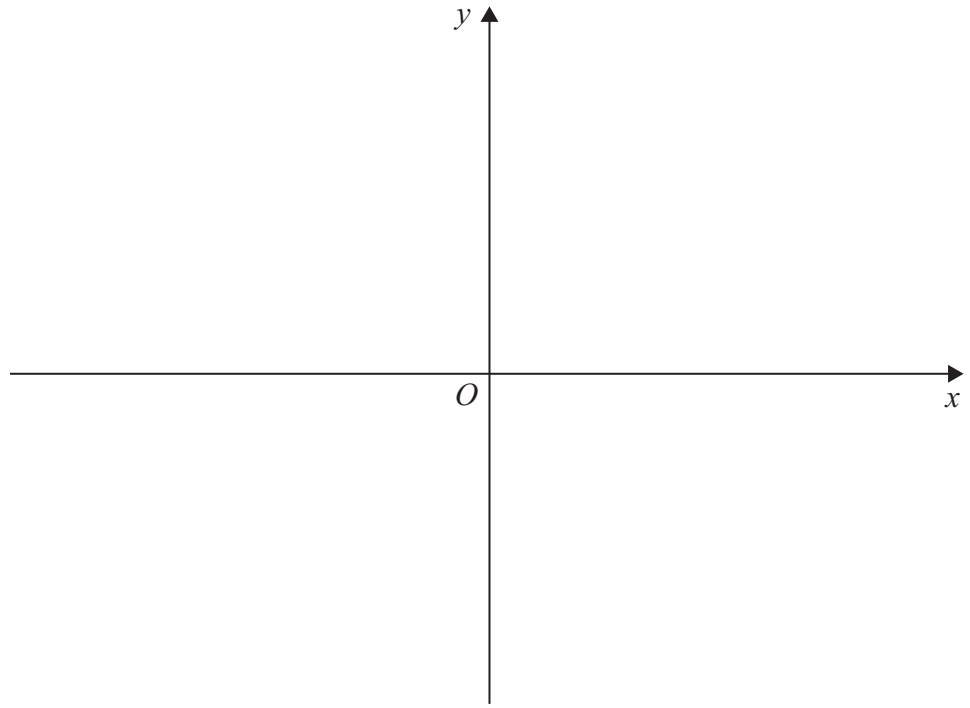
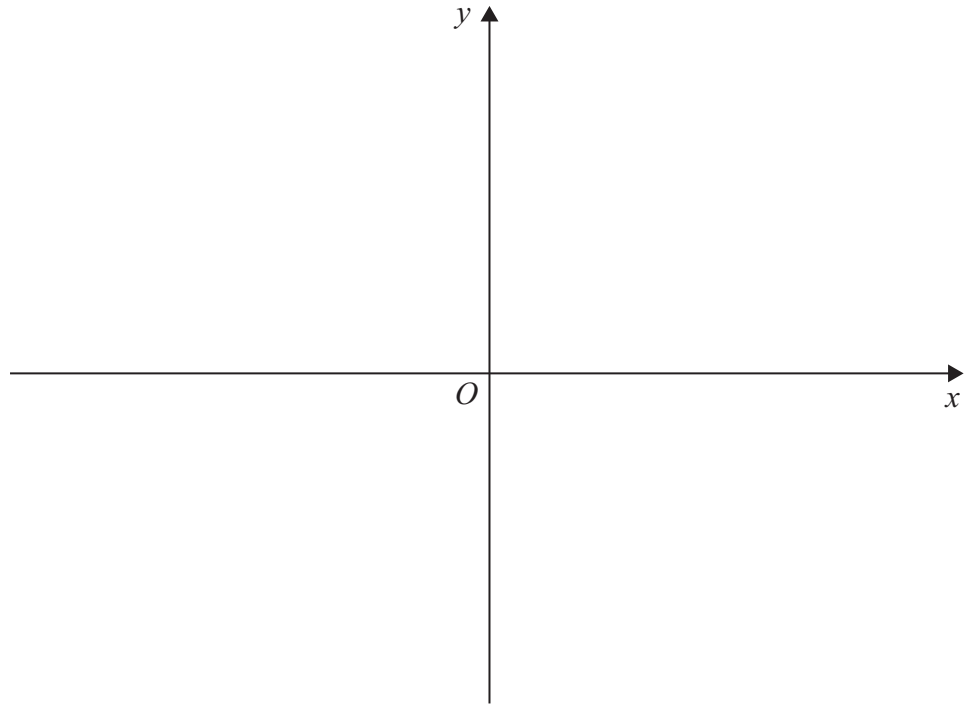
6 (b) Sketch the graph of $y = 2f(x) - 4$

[2 marks]



6 (c) Sketch the graph of $y = f'(x)$

[3 marks]



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7 (a) Using ${}^nC_r = \dfrac{n!}{r!(n-r)!}$ show that ${}^nC_2 = \dfrac{n(n-1)}{2}$

[2 marks]

7 (b) (i) Show that the equation

$$2 \times {}^nC_4 = 51 \times {}^nC_2$$

simplifies to

$$n^2 - 5n - 300 = 0$$

[3 marks]

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7 (b) (ii) Hence, solve the equation

$$2 \times {}^{n}C_{4} = 51 \times {}^{n}C_{2}$$

[2 marks]

Turn over for the next question

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8 The sum to infinity of a geometric series is 96

The first term of the series is less than 30

The second term of the series is 18

8 (a) Find the first term and common ratio of the series.

[5 marks]

8 (b) (i) Show that the nth term of the series, u_n, can be written as

$$u_n = \frac{3^n}{2^{2n-5}}$$

[4 marks]

8 (b) (ii) Hence show that

$$\log_3 u_n = n(1 - 2\log_3 2) + 5\log_3 2$$

[3 marks]

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9 (a) For $\cos\theta \neq 0$, prove that

$$\operatorname{cosec} 2\theta + \cot 2\theta = \cot\theta$$

[4 marks]

9 (b) Explain why

$$\cot\theta \neq \operatorname{cosec} 2\theta + \cot 2\theta$$

when $\cos\theta = 0$

[1 mark]

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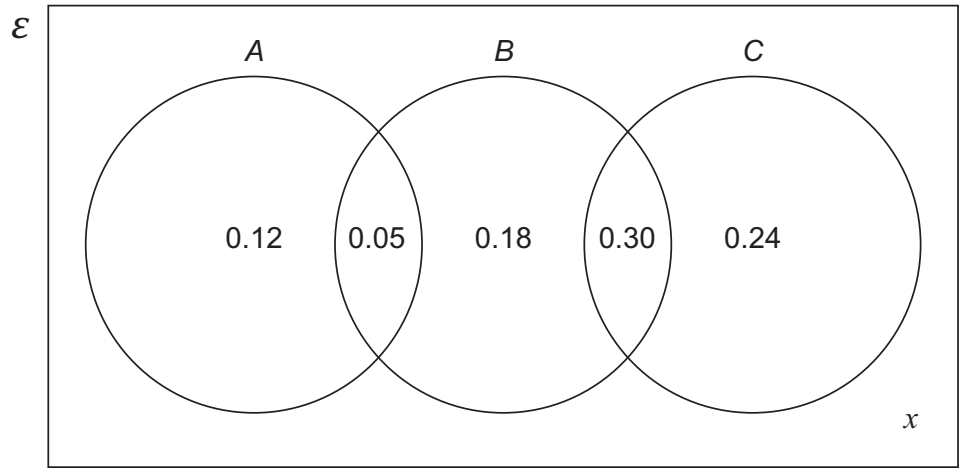
Section B

Answer **all** questions in the spaces provided.

10 The probabilities of events *A*, *B* and *C* are related, as shown in the Venn diagram below.



Find the value of x.

Circle your answer.

[1 mark]

0.11 0.46 0.54 0.89

11 The table below shows the temperature on Mount Everest on the first day of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperature (°C)	−17	−16	−14	−9	−2	2	6	5	−3	−4	−11	−18

Calculate the standard deviation of these temperatures.

Circle your answer.

[1 mark]

−6.75 5.82 8.24 67.85

Turn over ▶

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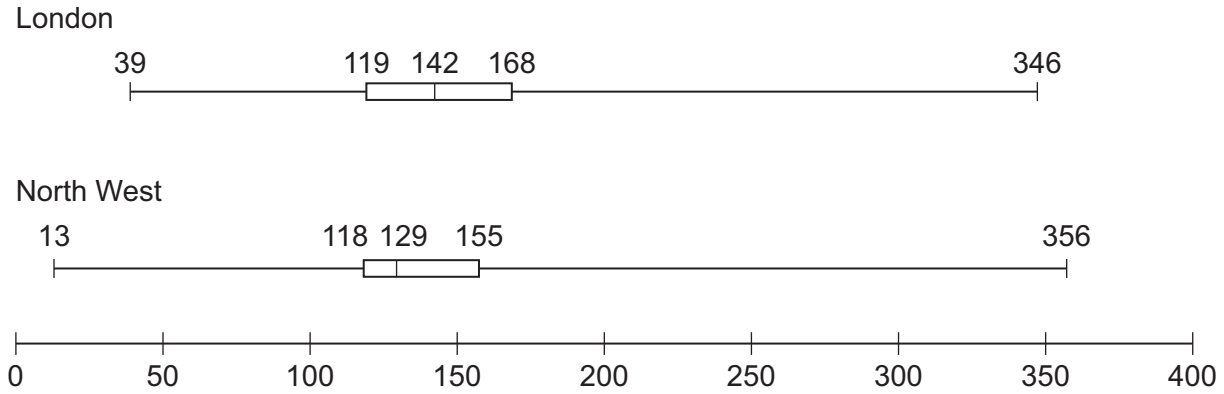
12 The box plot below summarises the CO_2 emissions, in g/km, for cars in the Large Data Set from the London and North West regions.



12 (a) Using the box plot, give **one** comparison of central tendency and **one** comparison of spread for the two regions.

[2 marks]

Comparison of central tendency

Comparison of spread

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12 (b) Jaspal, an environmental researcher, used all of the data in the Large Data Set to produce a statistical comparison of the CO_2 and CO emissions in regions of England.

Using your knowledge of the Large Data Set, give **two** reasons why his conclusions may be invalid.

[2 marks]

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Turn over for the next question

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13 Diedre is a head teacher in a school which provides primary, secondary and sixth-form education.

There are 200 teachers in her school.

The number of teachers in each level of education along with their gender is shown in the table below.

	Primary	**Secondary**	**Sixth-form**
Male	9	24	23
Female	35	85	24

13 (a) A teacher is selected at random. Find the probability that:

13 (a) (i) the teacher is female

[1 mark]

13 (a) (ii) the teacher is **not** a sixth-form teacher.

[1 mark]

13 (b) Given that a randomly chosen teacher is male, find the probability that this teacher is **not** a primary teacher.

[2 marks]

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13 (c) Diedre wants to select three different teachers at random to be part of a school project.

Calculate the probability that all three chosen are secondary teachers.

[2 marks]

Turn over for the next question

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14 It is known that a hospital has a mean waiting time of 4 hours for its Accident and Emergency (A&E) patients.

After some new initiatives were introduced, a random sample of 12 patients from the hospital's A&E Department had the following waiting times, in hours.

4.25 3.90 4.15 3.95 4.20 4.15

5.00 3.85 4.25 4.05 3.80 3.95

Carry out a hypothesis test at the 10% significance level to investigate whether the mean waiting time at this hospital's A&E department has changed.

You may assume that the waiting times are normally distributed with standard deviation 0.8 hours.

[7 marks]

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15 A political party is holding an election to choose a new leader.

A statistician within the party decides to sample 70 party members to find their opinions of the leadership candidates.

There are 4735 members under 30 years old and 8565 members 30 years old and over.

The statistician wants to use a sample of 70 party members in the survey.

He decides to use a random stratified sample.

15 (a) Calculate how many of each age group should be included in his sample.

[2 marks]

15 (b) Explain how he could collect the random sample of members under 30 years old.

[3 marks]

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16 An educational expert found that the correlation coefficient between the hours of revision and the scores achieved by 25 students in their A-level exams was 0.379

Her data came from a bivariate normal distribution.

Carry out a hypothesis test at the 1% significance level to determine if there is a positive correlation between the hours of revision and the scores achieved by students in their A-level exams.

The critical value of the correlation coefficient is 0.4622

[4 marks]

Turn over for the next question

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17 The lifetime of Zaple smartphone batteries, X hours, is normally distributed with mean 8 hours and standard deviation 1.5 hours.

17 (a) (i) Find $P(X \neq 8)$

[1 mark]

17 (a) (ii) Find $P(6 < X < 10)$

[1 mark]

17 (b) Determine the lifetime exceeded by 90% of Zaple smartphone batteries.

[2 marks]

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17 (c) A different smartphone, Kaphone, has its battery's lifetime, Y hours, modelled by a normal distribution with mean 7 hours and standard deviation σ.

25% of randomly selected Kaphone batteries last less than 5 hours.

Find the value of σ, correct to three significant figures.

[4 marks]

Turn over for the next question

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18 Tiana is a quality controller in a clothes factory. She checks for four possible types of defects in shirts.

Of the shirts with defects, the proportion of each type of defect is as shown in the table below.

Type of defect	Colour	Fabric	Sewing	Sizing
Probability	0.25	0.30	0.40	0.05

Shirts with defects are packed in boxes of 30 at random.

18 (a) Find the probability that:

18 (a) (i) a box contains exactly 5 shirts with a colour defect

[2 marks]

18 (a) (ii) a box contains fewer than 15 shirts with a sewing defect

[2 marks]

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18 (a) (iii) a box contains at least 20 shirts which do **not** have a fabric defect.

[3 marks]

Question 18 continues on the next page

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18 (b) Tiana wants to investigate the proportion, p, of defective shirts with a fabric defect.

She wishes to test the hypotheses

$$H_0 : p = 0.3$$

$$H_1 : p < 0.3$$

She takes a random sample of 60 shirts with a defect and finds that x of them have a fabric defect.

18 (b) (i) Using a 5% level of significance, find the critical region for x.

[5 marks]

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18 (b) (ii) In her sample she finds 13 shirts with a fabric defect.

Complete the test stating her conclusion in context.

[2 marks]

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