Centre Number   Candidate Number

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/4C**

Further Mathematics

Advanced
PAPER 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \, \text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 7 2 1 1 4 A 0 1 3 2

Pearson

1. Three particles of masses $2m$, $3m$ and km are placed at the points with coordinates $(3a, 2a)$, $(a, -4a)$ and $(-3a, 4a)$ respectively.

   The centre of mass of the three particles lies at the point with coordinates $(\overline{x}, \overline{y})$.

   (a) (i) Find $\overline{x}$ in terms of a and k

       (ii) Find $\overline{y}$ in terms of a and k

   (4)

   Given that the distance of the centre of mass of the three particles from the point $(0, 0)$ is $\dfrac{1}{3}a$

   (b) find the possible values of k

   (2)

(Total for Question 1 is 6 marks)

2. A cyclist and her cycle have a combined mass of 60 kg. The cyclist is moving along a straight horizontal road and is working at a constant rate of 200 W.

When she has travelled a distance x metres, her speed is $v\,\text{ms}^{-1}$ and the magnitude of the resistance to motion is $3v^2\,\text{N}$.

(a) Show that $\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{200 - 3v^3}{60v^2}$

(4)

The distance travelled by the cyclist as her speed increases from $2\,\text{ms}^{-1}$ to $4\,\text{ms}^{-1}$ is D metres.

(b) Find the exact value of D

(3)

Question 2 continued

(Total for Question 2 is 7 marks)

3.

Figure 1

Nine uniform rods are joined together to form the rigid framework *ABCDEFA*, with
AB = *BC* = *DF* = 3*a*, *BF* = *CD* = *DE* = 4*a* and *AF* = *FE* = *CF* = 5*a*, as shown in
Figure 1. All nine rods lie in the same plane.

The mass per unit length of each of the rods *BF*, *CF* and *DF* is twice the mass per unit
length of each of the other six rods.

(a) Find the distance of the centre of mass of the framework from *AC*

(4)

The mass of the framework is *M*. A particle of mass *kM* is attached to the framework at
E to form a loaded framework.

When the loaded framework is freely suspended from *F*, it hangs in equilibrium with
CE horizontal.

(b) Find the exact value of *k*

(3)

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Question 3 continued

Question 3 continued

(Total for Question 3 is 7 marks)

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4.

Figure 2

A small smooth ring R of mass m is threaded onto a light inextensible string. One end of the string is attached to a fixed point A and the other end of the string is attached to the fixed point B such that B is vertically above A and $AB = 6a$

The ring moves with constant angular speed ω in a horizontal circle with centre A. The string is taut and BR makes a constant angle θ with the downward vertical, as shown in Figure 2.

The ring is modelled as a particle.

Given that $\tan\theta = \dfrac{8}{15}$

(a) find, in terms of m and g, the magnitude of the tension in the string,

(3)

(b) find ω in terms of a and g

(5)

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Question 4 continued

(Total for Question 4 is 8 marks)

5.

Figure 3

The uniform plane lamina shown in Figure 3 is formed from two squares, *ABCO* and *ODEF*, and a sector *ODC* of a circle with centre *O*. Both squares have sides of length $3a$ and *AO* is perpendicular to *OF*. The radius of the sector is $3a$

[*In part* (a) *you may use, without proof, any of the centre of mass formulae given in the formulae booklet*.]

(a) Show that the distance of the centre of mass of the sector *ODC* from *OC* is $\dfrac{4a}{\pi}$

(3)

(b) Find the distance of the centre of mass of the lamina from *FC*

(4)

The lamina is freely suspended from *F* and hangs in equilibrium with *FC* at an angle $\theta°$ to the downward vertical.

(c) Find the value of θ

(4)

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(Total for Question 5 is 11 marks)

6.

Figure 4

The shaded region shown in Figure 4 is bounded by the x-axis, the line with equation $x = 9$ and the line with equation $y = \dfrac{1}{3}x$. This shaded region is rotated through $360°$ about the x-axis to form a solid of revolution. This solid of revolution is used to model a solid right circular cone of height $9\,\text{cm}$ and base radius $3\,\text{cm}$.

The cone is non-uniform and the mass per unit volume of the cone at the point (x, y, z) is $\lambda x\,\text{kg}\,\text{cm}^{-3}$, where $0 \leqslant x \leqslant 9$ and λ is constant.

(a) Find the distance of the centre of mass of the cone from its vertex.

(6)

A toy is made by joining the circular plane face of the cone to the circular plane face of a uniform solid hemisphere of radius $3\,\text{cm}$, so that the centres of the two plane surfaces coincide.

The weight of the cone is W newtons and the weight of the hemisphere is kW newtons.

When the toy is placed on a smooth horizontal plane with any point of the curved surface of the hemisphere in contact with the plane, the toy will remain at rest.

(b) Find the value of k

(4)

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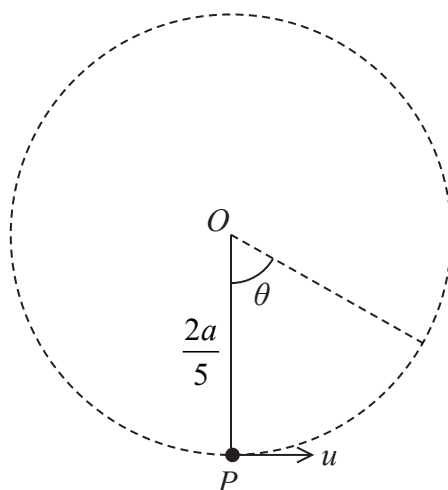(Total for Question 6 is 10 marks)

7.

Figure 5

A package P of mass m is attached to one end of a string of length $\dfrac{2a}{5}$. The other end of the string is attached to a fixed point O. The package hangs at rest vertically below O with the string taut and is then projected horizontally with speed u, as shown in Figure 5.

When OP has turned through an angle θ and the string is still taut, the tension in the string is T

The package is modelled as a particle and the string as being light and inextensible.

(a) Show that $T = 3mg\cos\theta - 2mg + \dfrac{5mu^2}{2a}$

(6)

Given that P moves in a complete vertical circle with centre O

(b) find, in terms of a and g, the minimum possible value of u

(2)

Given that $u = 2\sqrt{ag}$

(c) find, in terms of g, the magnitude of the acceleration of P at the instant when OP is horizontal.

(3)

(d) Apart from including air resistance, suggest one way in which the model could be refined to make it more realistic.

(1)

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(Total for Question 7 is 12 marks)

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8. **Throughout this question, use $g = 10\,\text{m}\,\text{s}^{-2}$**

A light elastic string has natural length $1.25\,\text{m}$ and modulus of elasticity $25\,\text{N}$.

A particle P of mass $0.5\,\text{kg}$ is attached to one end of the string. The other end of the string is attached to a fixed point A. Particle P hangs freely in equilibrium with P vertically below A

The particle is then pulled vertically down to a point B and released from rest.

(a) Show that, while the string is taut, P moves with simple harmonic motion with period $\dfrac{\pi}{\sqrt{10}}$ seconds.

(6)

The maximum kinetic energy of P during the subsequent motion is $2.5\,\text{J}$.

(b) Show that $AB = 2\,\text{m}$

(3)

The particle returns to B for the first time T seconds after it was released from rest at B

(c) Find the value of T

(5)

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Question 8 continued

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(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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