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Examiners' report

FURTHER MATHEMATICS A

H245

For first teaching in 2017

Y545/01 Summer 2022 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers are also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

Advance Information for Summer 2022 assessments

To support student revision, advance information was published about the focus of exams for Summer 2022 assessments. Advance information was available for most GCSE, AS and A Level subjects, Core Maths, FSMQ, and Cambridge Nationals Information Technologies. You can find more information on our website.

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Paper Y545/01 series overview

This summer was the first assessment of Y545 to run since 2019 and there was an entry of around 450 candidates. Considering the disruption of the previous two years due to Covid, the quality of the responses received were very promising indeed. Scoring 50 marks or more (out of 75) represented a considerable achievement and this was accomplished by over a quarter of the cohort. At the other end of the scale, less than 10% of the candidates did not pick up at least 20 marks.

Although greater detail will follow below in the comments on individual questions, there are a number of broad areas to be mentioned for the benefit of future candidates. As in past series, questions requiring explanation, justification or supporting reasoning were generally handled variably, with many candidates seemingly adopting the approach of constructing excessively lengthy responses in the hope that some of their words would earn the mark(s) allocated. Within the 75-minute time frame for this paper, it should be clear that written comments need to be brief and to the point; paragraphs of essay-style detail are rarely productive. Furthermore, a significant number of candidates struggled to cover the key point(s) within a written response; many candidates often managed to say something correct but were unable to state convincing mathematical reasons for their observation(s) and so could not be given (full) credit. This was especially so in Q4(a)(ii) and (iii) (number theory) and Q8 (groups) as well as to a lesser extent in Q5 (a) and Q6 (b) (i).

Candidates who did well on this paper generally did the following:	Candidates who did less well on this paper generally did the following:
• they were able to use and apply standard techniques with confidence • they did well in Questions 2 and 7b, on topics which have been regularly assessed in the past and could then be practised via past papers • they were able to apply their problem-solving skills to translate problems in both mathematical and non-mathematical contexts into mathematical processes (see Questions 6 and 9, respectively).	• they made arithmetic / algebraic errors which prevented them from gaining the easiest marks in the paper (for example in Questions 1, 4, 5) • they struggled to correctly start and carry out proofs of all kinds (for example in Questions 3, 7, 8) • they struggled to deal with abstract groups defined by their algebraic properties.

Question 2 (a)

2 Consider the integers a and b, where, for each integer n, $a = 7n + 4$ and $b = 8n + 5$.

Let $h = \text{hcf}(a, b)$.

(a) Determine all possible values of h. **[3]**

Approximately three quarters of candidates used the result that h|a and h|b implies that h|(ax+by) for any integers x and y and over a half of those chose the correct linear combination. However, a significant number of candidates did not use the divisibility notation, concluding that *h*=3 or did not reach the final solution. A quarter of the students started to use the Euclidean Algorithm to find the highest common factor, but very few of those managed to complete their solution.

This question is assessing two of AO2 strands, construct rigorous mathematical arguments (including proofs) and make deductions. It is therefore essential that candidates present their reasoning in a rigorous manner to obtain full marks.

Exemplar 1

2(a)	if $\text{hcf}(7n+4, 8n+5) = h$
	$h \mid A(7n+4) + B(8n+5)$
	$A = 8 \quad B = -7$
	$h \mid 56n + 32 - 56n - 35$
	$h \mid -3$
	$\therefore h = 3$ or $h = 1$

This response gains full marks. The candidate chooses a correct linear combination of the two given numbers and uses the correct notation, stating that h (the HCF) has to be a factor of such combination. Finally, they write down the two possible values of h.

Question 2 (b)

(b) Find all values of n for which a and b are **not** co-prime. **[2]**

Less than a half of candidates reached the correct solution. Many tried to find *n* by trial and error rather than solving a linear congruence.

Assessment for learning

Candidates would benefit from solving problems in which linear congruences are an efficient method to find a class of values for an integer.

Misconception

A significant number of candidates thought that h=ax+by rather than h|(ax+by).

Question 3

3 The irrational number $\phi = \frac{1}{2}(1+\sqrt{5})$ plays a significant role in the sequence of Fibonacci numbers given by $F_0 = 0$, $F_1 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for $n \geqslant 1$.

Prove by induction that, for each positive integer n, $\phi^n = F_n \times \phi + F_{n-1}$. **[6]**

Most candidates were able to gain the first 2 marks in this induction question. Less than a half of the candidates made enough progress with the proof to achieve most of the marks. Some candidates worked backwards from the final result rather than following the desired formal structure of the proof. There were also candidates who were not using the induction hypothesis (assumption of the truth of the statement for *n*=*k*) to prove that the statement holds for *n*=*k*+1 and they therefore fail to complete the proof. A small proportion of candidates decided to use strong induction, showing that P(1) and P(2) is true and that P(*k*) and P(*k*+1) true implies P(*k*+2) true as well for some *k* > 2. However, only a negligible number of responses gained full marks via strong induction.

In this question a large number of strands of AO2 are assessed (make deductions, explain reasoning, use mathematical language and notation correctly), together with one strand of AO3 (translate problems in mathematical contexts into mathematical processes).

Assessment for learning

Strong induction should only be used when strictly required, as it may make a proof unnecessarily complicated.

Question 4 (a) (i)

4 Let N be the number 15 824 578.

(a) (i) Use a standard divisibility test to show that N is a multiple of 11. **[2]**

The majority of candidates gained full marks on this question. Given that AO2 (explain reasoning) was assessed, an explanation that the sum of digits at odd places subtracted from the sum of digits at even places is a multiple of 11 was required.

Misconception

Some candidates wrote in their conclusion that since '0|11', N is a multiple of 11, thereby losing the final mark.

Question 4 (a) (ii)

(ii) A student uses the following test for divisibility by 7.

> 'Throw away' multiples of 7 that appear either individually or within a pair of consecutive digits of the test number.
> Stop when the number obtained is 0, 1, 2, 3, 4, 5 or 6.
> The test number is only divisible by 7 if that obtained number is 0.

For example, for the number N, they first 'throw away' the "7" in the tens column, leaving the number $N_1 = 15824508$. At the second stage, they 'throw away' the "14" from the left-hand pair of digits of N_1, leaving $N_2 = 01824508$; and so on, until a number is obtained which is 0, 1, 2, 3, 4, 5 or 6.

- Justify the validity of this process.
- Continue the student's test to show that $7 \mid N$. **[2]**

The 'Throw away' test was generally performed correctly, with just a few candidates demonstrating a misconception of the idea that they needed to 'throw away' a multiple of 7 which is also a multiple of ten. A few students were incorrectly removing a multiple of 7 from non-adjacent digits. The second part of the question tested AO2 - assess the validity of mathematical arguments. Many candidates found it hard to explain why this process is valid, failing to communicate that divisibility of a number by 7 is unchanged when subtracting multiples of 7 from the number.

Question 4 (a) (iii)

(iii) Given that $N = 11 \times 1438598$, explain why $7 \mid 1438598$. **[1]**

This question was generally well answered, but many candidates neglected to include that hcf(7,11)=1 in their reasoning.

Assessment for learning

It is important to justify that Euclid's Lemma holds since hcf(7,11)=1 (or equivalent explanation in words). There will generally be a mark for this observation, even though it may only be tested once (as this year, where it was required in this question and not in the more routine Q2(b)).

Question 4 (b) (i), (ii). (iii)

(b) Let $M = N^2$.

(i) Express N in the unique form $101a + b$ for positive integers a and b, with $0 \leqslant b < 101$. **[2]**

(ii) Hence write M in the form $M \equiv r \pmod{101}$, where $0 < r < 101$. **[1]**

(iii) Deduce the order of N modulo 101. **[1]**

Part (b) of Question 4 was generally well answered. In part (b)(i), the only problems occurred when candidates did not directly perform a division by 101 but tried to use more complex methods. Most of those who solved this part correctly, also produced the correct answer to part (b)(ii). Fewer than a half of the candidates answered part (b)(iii) correctly, demonstrating knowledge of the order of an integer modulo *n*.

Question 5 (a)

5 You are given the variable point $A(3, -8, t)$, where t is a real parameter, and the fixed point $B(1, 2, -2)$.

(a) Using only the geometrical properties of the vector product, explain why the statement "$\overrightarrow{OA} \times \overrightarrow{OB} = \mathbf{0}$" is false for all values of t. **[2]**

In this question, most candidates stated that a vector product of two non-zero vectors is zero when the vectors are parallel or collinear, thereby gaining the first mark. However, some neglected to provide a numerical argument to show that the two vectors cannot be multiples of each other, which was required by the AO2 (explain their reasoning) tested in this question.

Question 6 (a)

6 In a national park, the number of adults of a given species is carefully monitored and controlled. The number of adults, n months after the start of this project, is A_n. Initially, there are 1000 adults. It is predicted that this number will have declined to 960 after one month.

The first model for the number of adults is that, from one month to the next, a fixed proportion of adults is lost. In order to maintain a fixed number of adults, the park managers "top up" the numbers by adding a constant number of adults from other parks at the end of each month.

(a) Use this model to express the number of adults as a first-order recurrence system. **[1]**

In this question AO3 is assessed (translate situations in context into mathematical models). The majority of the candidates obtained the recurrence relation correctly, but did not write the first term, therefore gaining no mark, as they were asked to write a recurrence system. Some candidates wrote $A_0 = 1000$, $A_1 = 960$, which shows that they did not observe that the sequence is constant.

Question 6 (b) (i)

Instead, it is found that, the proportion of adults lost each month is double the predicted amount, with no change being made to the constant number of adults added each month.

(b) (i) Show that the revised recurrence system for A_n is $A_0 = 1000$, $A_{n+1} = 0.92A_n + 40$. **[1]**

Although most of the candidates observed that there are 8% of adults lost, they did not clearly link it to the multiplier 0.92. There were very few who made the observation that 1000 and 40 remain the same in the revised system.

Question 6 (b) (ii)

(ii) Solve this revised recurrence system. **[4]**

In this question only AO1 (use and apply standard techniques) was assessed. This was a well performed question with responses which were professionally presented.

Question 6 (b) (iii)

(iii) Describe the long-term behaviour of the sequence $\{A_n\}$ in this case. **[1]**

In this question AO3 (use mathematical models) was tested. Although most of the candidates observed that number of adults converges to the correct value, they tended not to comment on the fact that it decreases to 500.

Question 6 (c) (i)

A more refined model for the number of adults uses the second-order recurrence system $A_{n+1} = 0.9A_n - 0.1A_{n-1} + 50$, for $n \geqslant 1$, with $A_0 = 1000$ and $A_1 = 920$.

(c) **(i)** Determine the long-term behaviour of the sequence $\{A_n\}$ for this more refined model. **[4]**

In this question a strand of AO3, translate problems in mathematical and non-mathematical contexts into mathematical processes, was tested. Candidates were expected to state that the general solution is of the form $A_n = D \times \alpha^n + E \times \beta^n + 250$, where α and β are positive and less than 1. However, many went into lengthy calculations to find the values of all the constant involved, failing to realise that these are immaterial to the long-term behaviour of the sequence. While the vast majority of the candidates correctly calculated the particular solution and solved the auxiliary equation correctly, a common omission was not to comment on the behaviour of α^n and β^n (required by AO2 – explain their reasoning). Some wrote that $\alpha, \beta \to 0$, failing to observe that these are numbers.

Assessment for learning

Candidates would benefit from knowing that the long-term behaviour of sequences described by first- or second-order recurrence relations is determined by the size of the absolute value of the solutions to the auxiliary equation.

Question 6 (c) (ii)

(ii) A criticism of this more refined model is that it does not take account of the fact that the number of adults must be an integer at all times.

State a modified form of the second-order recurrence relation for this more refined model that will satisfy this requirement. **[1]**

In this question AO3, explain how to refine models, was assessed.

About a half of the candidates could provide a formula using one of the functions INT, floor or ceiling. Among those who did not answer correctly, the most common error was to multiply the recurrence relation by 10 or 100, failing to observe that this may only solve the problem if a decimal has 1dp or 2dp, respectively.

Question 7 (a)

7 **(a)** Differentiate $(16+t^2)^{\frac{3}{2}}$ with respect to t. **[1]**

Question 7 (b)

Let $I_n = \int_0^3 t^n \sqrt{16+t^2}\,\mathrm{d}t$ for integers $n \geqslant 1$.

(b) Show that, for $n \geqslant 3$, $(n+2)I_n = 125 \times 3^{n-1} - 16(n-1)I_{n-2}$. **[5]**

Almost all candidates knew that integration by parts is the right approach. However, some confused which factor they needed to integrate and which one they needed to differentiate. If candidates gained the first method mark, they tended to complete the process successfully.

Assessment for learning

Candidates need to be reminded about the correct notation of integrals, which includes dx and limits.

Question 7 (c)

(c) The curve C is defined parametrically by $x = t^4 \cos t$, $y = t^4 \sin t$, for $0 \leqslant t \leqslant 3$. The length of C is denoted by L.

Show that $L = I_3$. (You are not required to evaluate this integral.) **[4]**

In this question, which assessed AO1 only (use and apply standard techniques), most of the candidates obtained full marks. Those who didn't, either could not apply the product rule for differentiation or incorrectly squared of the sum of two terms.

Question 8 (a)

8 **(a)** Explain why all groups of even order must contain at least one self-inverse element (that is, an element of order 2). **[2]**

This question proved very challenging for all candidates. A very common, incorrect answer was that Lagrange theorem *dictates* the existence of a subgroup of order two for a group of even order. It was seen that, once a candidate appreciated that inverses come in pairs, they were able to gain full marks.

Assessment for learning

Candidates should understand the difference between a sufficient condition and a necessary condition.

Misconception

Lagrange's theorem states that in a finite group, the order of any subgroup divides the order of the group. A common misconception is that every divisor of the order of a group is the order of some subgroup. This does not hold in general. The smallest example is A4 (the alternating group of degree 4), which has 12 elements but no subgroup of order 6.

Question 8 (b)

(b) Prove that any group, in which every (non-identity) element is self-inverse, is abelian. **[2]**

This question, in which AO3 is assessed, proved very challenging for all candidates. Most of the candidates provided a cycle argument proving that if ab=ba, then the group is abelian, not recognising that this is a definition.

Assessment for learning

Candidates should be exposed to questions on abstract groups defined by their algebraic properties. For example, they should be able to show that any group in which every element is self-inverse is abelian.

OCR support

Useful resources are contained in the Delivery guide 8.03 Groups.

Question 8 (c)

(c) A student believes that, if x and y are two distinct, non-identity, self-inverse elements of a group, then the element xy is also self-inverse.

The table shown here is the Cayley table for the non-cyclic group of order 6, having elements i, a, b, c, d and e, where i is the identity.

	i	a	b	c	d	e
i	i	a	b	c	d	e
a	a	i	d	e	b	c
b	b	e	i	d	c	a
c	c	d	e	i	a	b
d	d	c	a	b	e	i
e	e	b	c	a	i	d

By considering the elements of this group, produce a counter-example which proves that this student is wrong. **[2]**

A good number of candidates completed this question successfully. The most common error was not to mention in the counterexample that the elements selected are self-inverse. Once it was noted, some candidates used the Cayley table incorrectly. Other candidates confused the self-inverse property with commutativity.

Both marks available in this question were AO2 marks (Reason, interpret and communicate mathematically).

Assessment for learning

If we wish to prove that the statement P ⇒ Q is false, we have to find two elements of the group for which P is true and Q is false. It is therefore essential to state that P holds for the chosen elements.

Misconception

A significant number of candidates were reading the Cayley table incorrectly and thought that the element associated to *ab* lies at the intersection of the column headed by *a* and the row headed by *b*, rather than the other way round.

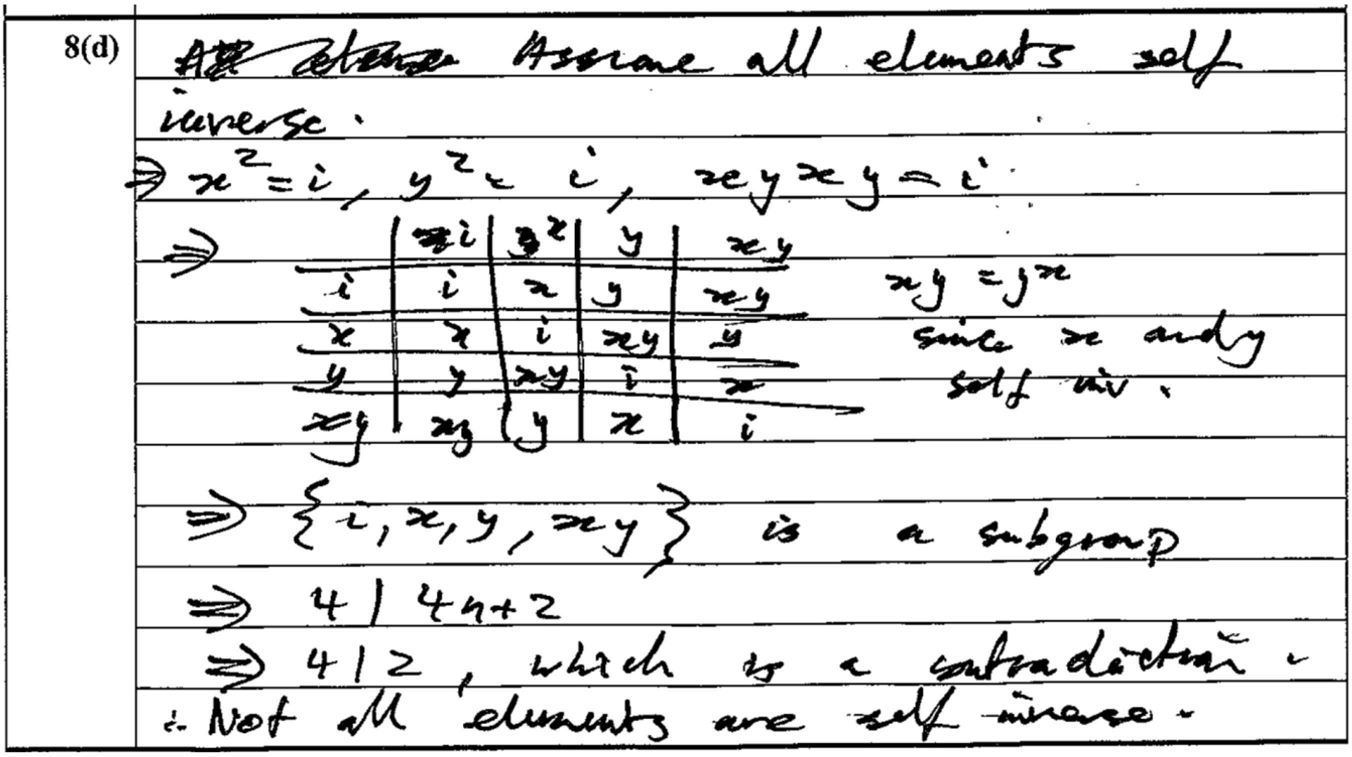
Question 8 (d)

(d) A group G has order $4n+2$, for some positive integer n, and i is the identity element of G. Let x and y be two distinct, non-identity, self-inverse elements of G. By considering the set $H = \{i, x, y, xy\}$, prove by contradiction that not all elements of G are self-inverse. **[4]**

This question proved very challenging for many candidates. Commonly scored marks were the first B and M marks, which shows that students generally know how to start a proof by contradiction and how Lagrange's theorem works. Many candidates demonstrated that H is a group (usually by means of a Cayley table) but did not explicitly state that H is a group or did not comment that it is a subgroup of G, thereby gaining no marks for their effort.

In this question, 3 out of the 4 marks available assessed AO2 (Reason, interpret and communicate mathematically). The question was meant to stretch and challenge, as reflected by its position in the question paper.

Exemplar 2



8(d) Assume all elements self inverse.

$\Rightarrow x^2 = i,\ y^2 = i,\ xyxy = i$

$\Rightarrow$

	i	x	y	xy
i	i	x	y	xy
x	x	i	xy	y
y	y	xy	i	x
xy	xy	y	x	i

$xy = yx$ since x and y self inv.

$\Rightarrow \{i, x, y, xy\}$ is a subgroup

$\Rightarrow 4 \mid 4n+2$

$\Rightarrow 4 \mid 2$, which is a contradiction.

$\therefore$ Not all elements are self inverse.

The main purpose of this exemplar is to indicate that it is not really necessary to write lengthy responses. This succinct response contains all the key elements to gain each of the 4 marks available: assumption that all (non-identity) elements of G are self-inverse, then H is a group and hence a subgroup of G, (Lagrange's theorem states that) the order of H divides the order of G, 4 is not a factor of (4n + 2), giving the required contradiction.

Exemplar 3

8(d)

the order of the set H is 4. 4 is not a factor of 4n+2 as 4n+2 = 2(2n+1) and 2n+1 is odd!
Thus, by Lagrange's theorem H cannot be a subgroup.
If xy was a self inverse then:

$(xy)^{-1} = (xy) \in G$. Also $(xy)^{-1} = y^{-1}x^{-1} = yx$

$x(xy) = (x^2)y = y \in G$

$(xy)x = (yx)x = yx^2 = y \in G$

$(xy)y = x(y^2) = x \in G$

$y(xy) = y(yx) = (y^2)x = x \in G$

$(x)(y) = xy \in G$

$(y)(x) = xy \in G$

Thus, H would be a subgroup as we have closure and inverse and this is contradiction

This candidate presents their arguments in a different order compared to the mark scheme. All the points have been addressed, except a clear assumption that all (non-identity) elements of G are self-inverse. The statement that 'If *xy* was a self-inverse' is not enough and therefore the first B mark is not gained and 3 marks were given

Question 9 (b)

(b) Determine the smallest value of the integer k for which the whole of S lies above the x-y plane. **[7]**

The purpose of part (a) was to lead candidates to consider z as a function of $2x + y$. Instead, it seemed to lead them towards the alternative method in the mark scheme, based on setting one or both partial derivatives equal to zero, determining x or y in terms of y or x, respectively and replacing one variable in z with the obtained expression. Almost all the candidates were able to score the first M mark. If they passed this point and continued on, then they were likely to obtain 5 or even 6 marks. In most cases, candidates did not justify that the minimum value of z occurs when $4x + 2y + 3 = 0$. They tried to prove it using the Hessian matrix, but few went on to observe that the function is convex, but not strictly convex (in their words). A well-commented graph (or graphs of sections) or a comment on the behaviour of z for very large x and y was sufficient, which the most able candidates provided. Very few candidates followed the main scheme, observing that z is a (convex, quadratic) function of $2x + y$. These candidates were more successful in scoring full marks.

In this stretch and challenge question, AO3 (interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations) and AO2 (construct rigorous mathematical arguments and explain reasoning) were assessed.

Exemplar 4

9(b)

$z =$ ~~$(2x+$~~ ~~xy~~ positive

$4x^2+4xy+y^2+6x+3y+k>0$

~~$(2x+y)^2 = (4x^2+y^2+4xy$~~

~~$(2x+y)^2 + 6x+3y+k>0$~~

~~$(y+\frac{3}{2})^2 - \frac{9}{4}+k$~~

$(2x+y)^2+6x+3y+k>0$

$(2x+y)^2+3(2x+y)+k>0$

let $(2x+y) = a$

$a^2+3a+k>0$

discriminant we want no solutions so discriminant <0

$3^2-4\times 1\times k<0$

$9-4k<0$

$\frac{9}{4}<k$

In this response the candidate follows the main mark scheme, identifying that z is a function of $2x + y$ and correctly stating that the discriminant of the quadratic this obtained must be negative for positivity in all the domain. The candidate gains 6 of the 7 marks available; the last mark is lost as the least integral value for integer k is not given.

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