OCR
Oxford Cambridge and RSA

Wednesday 6 October 2021 – Afternoon

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours

* 8 9 7 9 4 3 5 3 2 5 *

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)\big(\mathrm{f}(x)\big)^n\,\mathrm{d}x = \frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

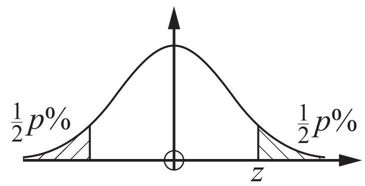
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (21 marks)

1 Beth states that for all real numbers p and q, if $p^2 > q^2$ then $p > q$.

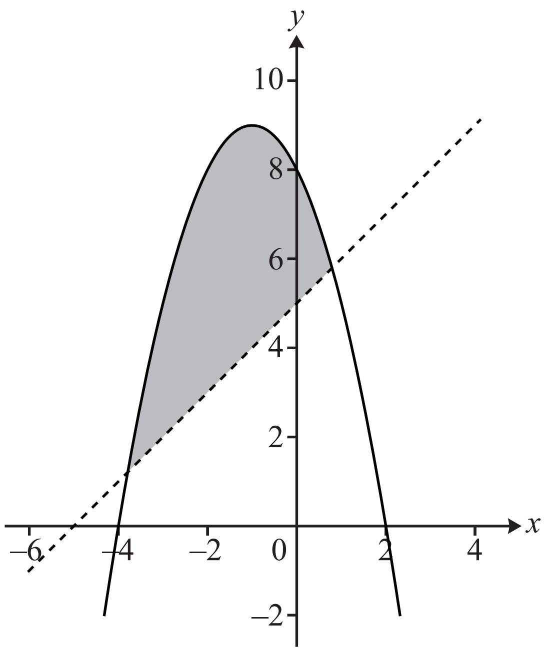
Prove that Beth is **not** correct. **[2]**

2 An unmanned spacecraft has a weight of 5200 N on Earth. It lands on the surface of the planet Mars where the acceleration due to gravity is $3.7\,\mathrm{m\,s^{-2}}$.

Calculate the weight of the spacecraft on Mars. **[3]**

3 **(a)** The diagram shows the line $y = x + 5$ and the curve $y = 8 - 2x - x^2$. The shaded region is the finite region between the line and the curve. The curved part of the boundary is included in the region but the straight part is not included.

Write down the inequalities that define the shaded region. **[2]**



(b) **In this question you must show detailed reasoning.**

Solve the inequality $8 - 2x - x^2 > x + 5$ giving your answer in exact form. **[3]**

4 **(a)** The first four terms of a sequence are 2, 3, 0, 3 and the subsequent terms are given by $a_{k+4} = a_k$.

(i) State what type of sequence this is. **[1]**

(ii) Find $\sum_{k=1}^{200} a_k$. **[1]**

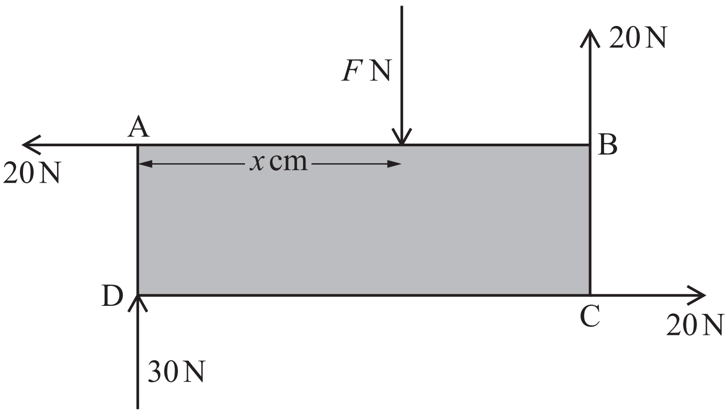
(b) A different sequence is given by $u_n = b^n$ where b is a constant and $n \geqslant 1$.

(i) State the set of values of b for which this is a divergent sequence. **[2]**

(ii) In the case where $b = \frac{1}{3}$, find the sum of all the terms in the sequence. **[2]**

5 ABCD is a rectangular lamina in which AB is 30 cm and AD is 10 cm.

Three forces of 20 N and one force of 30 N act along the sides of the lamina as shown in the diagram.



An additional force F N is also applied at right angles to AB to a point on the edge AB x cm from A.

(a) Given that the lamina is in equilibrium, calculate the values of F and x. **[3]**

The point of application of the force F N is now moved to B, but the magnitude and direction of the force remain the same.

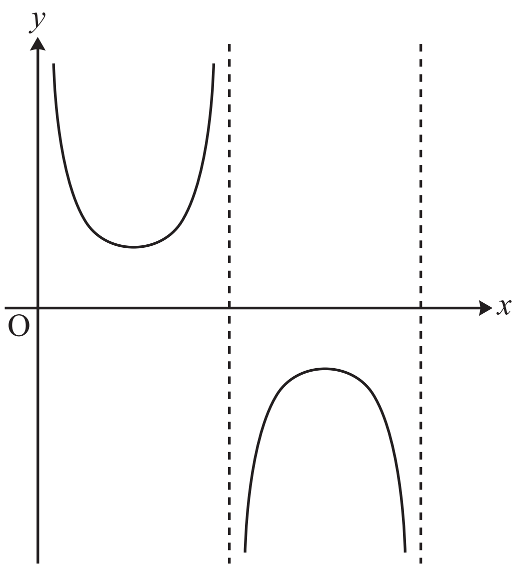
(b) Explain the effect of the new system of forces on the lamina. **[2]**

Answer **all** the questions.

Section B (79 marks)

6 **(a)** The diagram shows part of the graph of $y = \operatorname{cosec} x$, where x is in radians.

State the equations of the **three** vertical asymptotes that can be seen. **[1]**



The tangent to the graph at the point P with x-coordinate $\frac{\pi}{3}$ meets the x-axis at Q.

(b) Show that the x-coordinate of Q is $\frac{\pi}{3} + \sqrt{3}$. (You may use without proof the result that the derivative of $\operatorname{cosec} x$ is $-\operatorname{cosec} x \cot x$.) **[6]**

7 **In this question you must show detailed reasoning.**

The points A $(-1, 4)$ and B $(7, -2)$ are at opposite ends of a diameter of a circle.

(a) Find the equation of the circle. **[4]**

(b) Find the coordinates of the points of intersection of the circle and the line $y = 2x + 5$. **[3]**

(c) Q is the point of intersection with the larger y-coordinate.

Calculate the area of the triangle ABQ. **[3]**

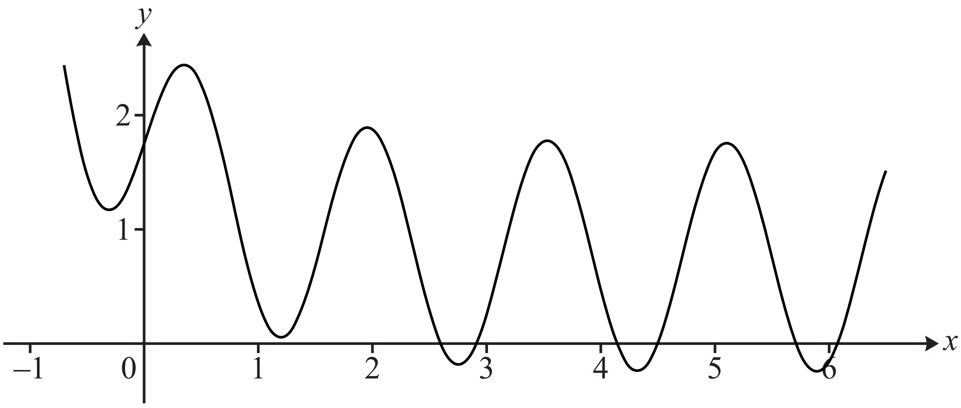
8 Kareem wants to solve the equation $\sin 4x + e^{-x} + 0.75 = 0$. He uses his calculator to create the following table of values for $f(x) = \sin 4x + e^{-x} + 0.75$.

x	0	1	2	3	4	5	6
$f(x)$	1.750	0.361	1.875	0.263	0.480	1.670	−0.153

He argues that because $f(6)$ is the first negative value in the table, there is a root of the equation between 5 and 6.

(a) Comment on the validity of his argument. **[1]**

The diagram shows the graph of $y = \sin 4x + e^{-x} + 0.75$.



(b) Explain why Kareem failed to find other roots between 0 and 6. **[1]**

Kareem decides to use the Newton-Raphson method to find the root close to 3.

(c) (i) Determine the iterative formula he should use for this equation. **[2]**

(ii) Use the Newton-Raphson method with $x_0 = 3$ to find a root of the equation $f(x) = 0$. Show **three** iterations and give your answer to a suitable degree of accuracy. **[3]**

Kareem uses the Newton-Raphson method with $x_0 = 5$ and also with $x_0 = 6$ to try to find the root which lies between 5 and 6. He produces the following tables.

x_0	5
x_1	3.97288
x_2	4.12125

x_0	6
x_1	6.09036
x_2	6.07110

(d) (i) For the iteration beginning with $x_0 = 5$, represent the process on the graph in the Printed Answer Booklet. **[2]**

(ii) Explain why the method has failed to find the root which lies between 5 and 6. **[2]**

(iii) Explain how Kareem can adapt his method to find the root between 5 and 6. **[1]**

9 The diagram shows a toy caterpillar consisting of a head and three body sections each connected by a light inextensible ribbon. The head has a mass of 120 g and the body sections each have a mass of 90 g.

The toy is pulled on level ground using a horizontal string attached to the head. The tension in the string is 12 N. There are resistances to motion of 2.5 N for the head and each section of the body.

(a) **(i)** State the equation of motion for the toy caterpillar modelled as a single particle. **[2]**

(ii) Calculate the acceleration of the toy caterpillar. **[1]**

(b) Draw a diagram showing all the forces acting on the head of the toy caterpillar. **[3]**

(c) Calculate the tension in the ribbon that joins the head to the body. **[2]**

10 A ball is thrown upwards with a velocity of $29.4\,\mathrm{m\,s^{-1}}$.

(a) Show that the ball reaches its maximum height after 3 s. **[1]**

(b) Sketch a velocity-time graph for the first 5 s of motion. **[2]**

(c) Calculate the speed of the ball 5 s after it is thrown. **[3]**

A second ball is thrown at $u\,\mathrm{m\,s^{-1}}$ at an angle of $\alpha°$ above the horizontal. It reaches the same maximum height as the first ball.

(d) Use this information to write down

- the vertical component of the second ball's initial velocity,
- the time taken for the second ball to reach its greatest height. **[2]**

This second ball reaches its greatest height at a point which is 48 m horizontally from the point of projection.

(e) Calculate the values of u and α. **[3]**

11 A balloon is being inflated. The balloon is modelled as a sphere with radius x cm at time t s. The volume $V\,\text{cm}^3$ is given by $V = \frac{4}{3}\pi x^3$.

The rate of increase of volume is inversely proportional to the radius of the balloon. Initially, when $t = 0$, the radius of the balloon is 5 cm and the volume of the balloon is increasing at a rate of $21\,\text{cm}^3\,\text{s}^{-1}$.

(a) Show that x satisfies the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{105}{4\pi x^3}$. **[5]**

(b) Find the radius of the balloon after two minutes. **[5]**

(c) Explain why the model may not be suitable for very large values of t. **[1]**

12 A box of mass m kg slides down a rough slope inclined at 15° to the horizontal. The coefficient of friction between the box and the slope is 0.4. The box has an initial velocity of $1.2\,\text{m}\,\text{s}^{-1}$ down the slope.

Calculate the distance the box travels before coming to rest. **[7]**

13 In this question $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x- and y-directions respectively.

The velocity of a particle at time t s is given by $(3t^2\mathbf{i} + 7\mathbf{j})\,\text{m}\,\text{s}^{-1}$. At time $t = 0$ the position of the particle with respect to the origin is $(-\mathbf{i} + 2\mathbf{j})$ m.

(a) Determine the distance of the particle from the origin when $t = 2$. **[6]**

(b) Show that the cartesian equation of the path of the particle is $x = \left(\frac{y-2}{7}\right)^3 - 1$. **[3]**

(c) At time $t = 2$, the magnitude of the resultant force acting on the particle is 48 N.

Find the mass of the particle. **[4]**

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