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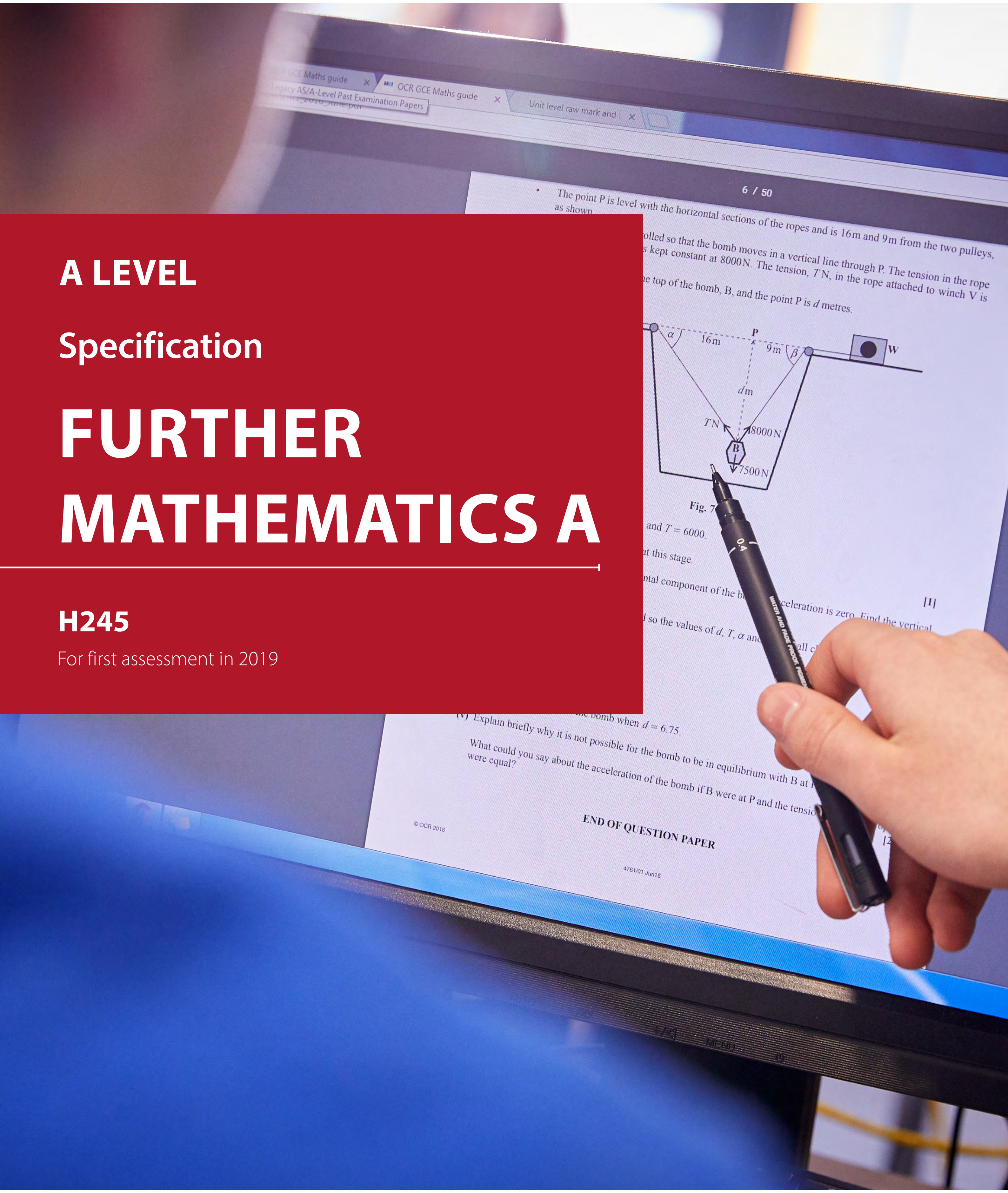

A LEVEL

Specification

FURTHER MATHEMATICS A

H245

For first assessment in 2019

Version 1.7 (February 2024)

ocr.org.uk/alevelfurthermaths

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Contents

1 Why choose an OCR A Level in Further Mathematics A?

1

1a. Why choose an OCR qualification?

Choose OCR and you've got the reassurance that you're working with one of the UK's leading exam boards. Our new A Level in Mathematics course has been developed in consultation with teachers, employers and Higher Education to provide learners with a qualification that's relevant to them and meets their needs.

We're part of the Cambridge Assessment Group, Europe's largest assessment agency and a department of the University of Cambridge. Cambridge Assessment plays a leading role in developing and delivering assessments throughout the world, operating in over 150 countries.

We work with a range of education providers, including schools, colleges, workplaces and other institutions in both the public and private sectors. Over 13,000 centres choose our A Levels, GCSEs and vocational qualifications including Cambridge Nationals and Cambridge Technicals.

Our Specifications

We believe in developing specifications that help you bring the subject to life and inspire your students to achieve more.

We've created teacher-friendly specifications based on extensive research and engagement with the teaching community. They're designed to be straightforward and accessible so that you can tailor the delivery of the course to suit your needs. We aim to encourage learners to become responsible for their own learning, confident in discussing ideas, innovative and engaged.

We provide a range of support services designed to help you at every stage, from preparation through to the delivery of our specifications. This includes:

- A wide range of high-quality creative resources including:
    - Delivery Guides
    - Transition Guides
    - Topic Exploration Packs
    - Lesson Elements
    - …and much more.
- Access to subject advisors to support you through the transition and throughout the lifetime of the specifications.
- CPD/Training for teachers including events to introduce the qualifications and prepare you for first teaching.
- Active Results – our free results analysis service to help you review the performance of individual learners or whole schools.
- ExamBuilder – our new free online past papers service that enables you to build your own test papers from past OCR exam questions can be found on the website at: www.ocr.org.uk/exambuilder

All A level qualifications offered by OCR are accredited by Ofqual, the Regulator for qualifications offered in England. The accreditation number for OCR's A Level in Further Mathematics A is QN603/1325/0

1b. Why choose an OCR A Level in Further Mathematics A?

OCR's A Level in Further Mathematics A is a coherent course of study that supports the development of mathematically informed individuals. It encourages learners to think and act mathematically, using mathematical skills and forms of communication to analyse situations within mathematics and elsewhere.

The course provides a balance between breadth and depth of mathematical knowledge. The pure core provides the foundations for further mathematical study, onto which learners add two options taken from Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. These options provide flexibility, allowing OCR's A Level in Further Mathematics to prepare students for further study and employment in a wide range of highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.

OCR's A Level in Further Mathematics A is designed for students who wish to study beyond an A Level in Mathematics, and provides a solid foundation for progression into further study particularly in mathematics, engineering, computer science, the sciences and economics.

OCR's A Level in Further Mathematics A is both broader and deeper than A Level Mathematics. A Level Further Mathematics A builds from GCSE (9–1) Mathematics and A Level Mathematics. As well as building on the algebra and calculus introduced in A Level Mathematics, the A Level Further Mathematics pure core content introduces complex numbers and matrices; fundamental mathematical ideas with wide applications in mathematics, engineering, physical sciences and computing. The non-core content includes different options that can enable learners to specialise in areas of mathematics that are particularly relevant to their interests and future aspirations, and gives learners the opportunity to extend their knowledge in applied mathematics and logical reasoning.

AS Level Further Mathematics A can be co-taught with A Level Further Mathematics A as a separate qualification. It consolidates and develops GCSE (9–1) Mathematics and AS Level Mathematics and supports transition to higher education or employment in any of the many disciplines that make use of quantitative analysis, including the social sciences, business, accounting and finance, mathematics, engineering, computer science, the sciences and economics.

This qualification is part of a wide range of OCR mathematics qualifications, allowing progression from Entry Level Certificate through GCSE to Core Maths, AS and A Level.

We appreciate that one size doesn't fit all so we offer two suites of qualifications in mathematics and further mathematics.

Mathematics A builds on our existing popular course. We've based the redevelopment of our current suite around an understanding of what works well in centres and have updated areas of content and assessment where stakeholders have identified that improvements could be made. We've undertaken a significant amount of consultation through our mathematics forums (which include representatives from learned societies, HE, teaching and industry) and through focus groups with teachers.

Mathematics B (MEI) has been developed in collaboration with Mathematics in Education and Industry, and is based on the existing suite of qualifications assessed by OCR. This is a well-established partnership which provides a firm foundation for curriculum and qualification development. MEI is a long established, independent curriculum development body; in developing Mathematics B (MEI), MEI has consulted with teachers and representatives from Higher Education to decide how best to meet the long term needs of learners.

All of our specifications have been developed with subject and teaching experts. We have worked in close consultation with teachers and representatives from Higher Education (HE).

Aims and learning outcomes

OCR's A Level in Further Mathematics A will encourage learners to:

- understand mathematics and mathematical processes in ways that promote confidence, foster enjoyment and provide a strong foundation for progress to further study
- extend their range of mathematical skills and techniques
- understand coherence and progression in mathematics and how different areas of mathematics are connected
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly
- reason logically and recognise incorrect reasoning
- generalise mathematically
- construct mathematical proofs
- use their mathematical skills and techniques to solve challenging problems which require them to decide on the solution strategy
- recognise when mathematics can be used to analyse and solve a problem in context
- represent situations mathematically and understand the relationship between problems in context and mathematical models that may be applied to solve them
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions
- make deductions and inferences and draw conclusions by using mathematical reasoning
- interpret solutions and communicate their interpretation effectively in the context of the problem
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding
- read and comprehend articles concerning applications of mathematics and communicate their understanding
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

1c. What are the key features of this specification?

The key features of OCR's A Level in Further Mathematics A for you and your learners are:

- a specification developed by teachers specifically for teachers, laying out the content clearly in terms of topic area, showing clear progression through the course and supporting co-teaching with A Level Mathematics A and AS Level Further Mathematics A.
- a simple assessment model featuring four papers of equal length and a free choice of two

options from four, so that learners can follow the most appropriate pathway for their interests and aspirations.

- a team of subject advisors, who can be contacted by centres for subject and assessment queries.

This specification is:

Worthwhile

- Research, international comparisons and engagement with both teachers and the wider education community have been used to enhance the reliability, validity and appeal of our assessment tasks in mathematics.
- It will encourage the teaching of interesting mathematics, aiming for mastery leading to positive exam results.

Learner-focused

- OCR's specification and assessment will consist of mathematics fit for the modern world and presented in authentic contexts.
- It will allow learners to develop mathematical independence built on a sound base of conceptual learning and understanding.
- OCR will target support and resources to develop fluency, reasoning and problem solving skills
- It will be a springboard for future progress and achievement in employment and in a variety of subjects in Higher Education.

Teacher-centred

- OCR will provide clear communication and an extensive teacher support package, including high-quality flexible resources, particularly for the new A Level Further Mathematics subject areas and to support the use of technology, proof, modelling and problem solving.
- OCR's support and resources will focus on empowering teachers, exploring teaching methods and classroom innovation alongside more direct content-based resources.
- OCR's assessment will be solid and dependable, recognising positive achievement in candidate learning and ability.

Dependable

- OCR's high-quality assessments are backed up by sound educational principles and a belief that the utility, richness and power of mathematics should be made evident and accessible to all learners.
- An emphasis on learning and understanding mathematical concepts underpinned by a sound, reliable and valid assessment.

1d. How do I find out more information?

If you are already using OCR specifications you can contact us at: www.ocr.org.uk

If you are not already a registered OCR centre then you can find out more information on the benefits of becoming one at: www.ocr.org.uk

If you are not yet an approved centre and would like to become one go to: www.ocr.org.uk

Want to find out more?

Get in touch with one of OCR's Subject Advisors:

Email: maths@ocr.org.uk

Customer Contact Centre: 01223 553998

Visit our Online Support Centre at support.ocr.org.uk

2 The specification overview

2a. OCR's A Level in Further Mathematics A (H245)

OCR's A Level in Further Mathematics A is a linear qualification in which all papers must be taken in the same examination series. All learners must take the two mandatory Pure Core papers Y540 and Y541 and any two* of the optional papers Y542, Y543, Y544 and Y545 to be awarded OCR's A Level in Further Mathematics A.

2

The subject content consists of a mandatory Pure Core and four optional areas: Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics.

The Overarching Themes must be applied along with associated mathematical thinking and understanding, across the whole of the subject content, see Section 2b.

*Learners may take more than two optional papers to increase the breadth of their course. For details of how their grade will be awarded, see Section 3g.

Content Overview	Assessment Overview	
Mandatory Pure Core All learners will study the content of the Pure Core. Papers Y540 and Y541 both assess content from the whole of the Pure Core and all of the Overarching Themes.	**Pure Core 1 (Y540)** 75 marks 90 minute written paper	**25%** of total A Level
	Pure Core 2 (Y541) 75 marks 90 minute written paper	**25%** of total A Level
Optional Papers Learners will study any two areas* chosen from Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. These papers assess the relevant content area and all of the Overarching Themes.	**Two of:** • **Statistics (Y542)** • **Mechanics (Y543)** • **Discrete Mathematics (Y544)** • **Additional Pure Mathematics (Y545)** Each: 75 marks 90 minute written paper	**25%** of total A Level
		25% of total A Level

2b. Content of A Level in Further Mathematics A (H245)

This A level qualification builds on the skills, knowledge and understanding set out in the whole GCSE subject content for mathematics for first teaching from 2015 and in the GCE A Level subject content for mathematics for first teaching from 2017. All of this content is assumed, but will only be explicitly assessed where it appears in this specification.

This is a linear qualification. The content is arranged by topic area and exemplifies the level of demand across two stages. The content for each area is shown in sections 2c-2g in two columns, demonstrating the progression across each topic. When this course is being co-taught with AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. Statements have a unique reference code. For ease of comparison, planning and co-teaching the 'Stage 1' content statements in this specification have reference codes corresponding to the same statements in OCR's AS Level in Further Mathematics A (H235). The content in these statements is identical, but the exemplification may differ as appropriate to the qualification.

In general, the content of A Level Further Mathematics 'Stage 1' only assumes knowledge from A Level Mathematics 'Stage 1'. Occasionally knowledge and skills from the content of A Level Mathematics 'Stage 2' which are not in A Level Further Mathematics 'Stage 1' are assumed; this is indicated in the relevant content statements.

The content is separated into five areas: Pure Core, Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. All learners must study the Pure Core and at least two of the optional areas. Centres are free to teach the content in the order most appropriate to their learners' needs.

Sections 4, 5, 6, 7 and 8 cover the pure core, statistics, mechanics, discrete mathematics and additional pure mathematics content of A Level Further Mathematics. In our mathematics specifications (H230 and H240) we have used the numbering 1, 2 and 3 to cover the pure mathematics, statistics and mechanics sections in order to facilitate the co-teaching of both qualifications.

The italic text in the content statements provides examples and further detail of the requirements of this specification. All exemplars contained in the specification under the heading "e.g." are for illustration only and do not constitute an exhaustive list. The heading "i.e." is used to denote a complete list. For the avoidance of doubt an italic statement in square brackets indicates content which will not be tested.

The expectation is that some assessment items will require learners to use two or more content statements without further guidance. Learners are expected to have explored the connections between their optional areas and the Pure Core. Learners may be required to demonstrate their understanding of the Pure Core content, and/or the content of A Level Mathematics (H240), within the optional papers, but that content will not be explicitly assessed.

Learners are expected to be able to use their knowledge to reason mathematically and solve problems both within mathematics and in context. Content that is covered by any statement may be required in problem solving, modelling and reasoning tasks even if that is not explicitly stated in the statement.

Problem solving, proof and mathematical modelling will be assessed in further mathematics in the context of the wider knowledge which students taking A Level further mathematics will have studied.

In **Pure Core (section 2c)** learners will extend and deepen their knowledge of proof, algebra, functions, calculus, vectors and differential equations studied in A Level Mathematics. They will also broaden their knowledge into other areas of pure mathematics that underpin the further study of mathematics and other numerate subjects with complex numbers, matrices, polar coordinates and hyperbolic functions.

In **Statistics (section 2d)** learners will explore the theory which underlies the statistics content in A Level Mathematics, as well as extending their tool box of statistical concepts and techniques. This area covers probability involving combinatorics, probability distributions for discrete and continuous random variables, hypothesis tests and confidence intervals for a population mean, χ-squared tests, non-parametric tests, correlation and regression.

In **Mechanics (section 2e)** learners extend their knowledge of particles, kinematics and forces from A Level Mathematics, using their extended pure mathematical knowledge to explore more complex physical systems. The area covers dimensional analysis, work, energy, power, impulse, momentum, centres of mass, circular motion and variable force.

Discrete Mathematics (section 2f) is the part of mathematics dedicated to the study of discrete objects. Learners will study both pure mathematical structures and techniques and their application to solving real-world problems of existence, construction, enumeration and optimisation. Areas studied include counting, graphs and networks, algorithms, critical path analysis, linear programming, and game theory.

In **Additional Pure Mathematics (section 2g)** learners will broaden and deepen their knowledge of pure mathematics, studying both discrete and continuous topics which form the foundation of undergraduate study in mathematics and mathematical disciplines. This area covers recurrence relations, number theory, group theory, the vector product, surfaces and partial differentiation.

Use of technology

It is assumed that learners will have access to appropriate technology when studying this course such as mathematical and statistical graphing tools and spreadsheets. When embedded in the mathematics classroom, the use of technology can facilitate the visualisation of certain concepts and deepen learners' overall understanding. The primary use of technology at this level is to offload computation and visualisation, to enable learners to investigate and generalise from patterns. Learners are not expected to be familiar with any particular software, but they are expected to be able to use their calculator for any function it can perform, when appropriate.

Suggested applications of technology to teaching and learning may be found in the introduction to each content area.

Use of calculators

Learners are permitted to use a scientific or graphical calculator for all papers. Calculators are subject to the rules in the document Instructions for Conducting Examinations, published annually by JCQ (www.jcq.org.uk).

It is expected that calculators available in the assessment will include the following features:

- An iterative function such as an ANS key.
- The ability to perform calculations, including inversion, with matrices up to at least order 3×3.
- The ability to compute summary statistics and access probabilities from the binomial and normal distributions.

Allowable calculators can be used for any function they can perform. When using calculators, candidates should bear in mind the following:

1. Candidates are advised to write down explicitly any expressions, including integrals, that they use the calculator to evaluate.

2. Candidates are advised to write down the values of any parameters and variables that they input into the calculator. Candidates are not expected to write down data transferred from question paper to calculator.

3. Correct mathematical notation (rather than "calculator notation") should be used; incorrect notation may result in loss of marks.

Formulae

Learners will be given a Formulae Booklet in each assessment, which includes the formulae given for OCR's A Level in Mathematics A. See section 5e for the content of this booklet.

Pre-release
There is no pre-released large data set for this qualification.

Simplifying expressions
It is expected that learners will simplify algebraic and numerical expressions when giving their final answers, even if the examination question does not explicitly ask them to do so. For example

- $80\frac{\sqrt{3}}{2}$ should be written as $40\sqrt{3}$,
- $\frac{1}{2}(1+2x)^{-\frac{1}{2}} \times 2$ should be written as either $(1+2x)^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{1+2x}}$,
- $\ln 2 + \ln 3 - \ln 1$ should be written as $\ln 6$,
- the equation of a straight line should be given in the form $y = mx + c$ or $ax + by = c$ unless otherwise stated.

The meanings of some instructions used in examination questions

In general, learners should show sufficient detail of their working to indicate that a correct method is being used. The following command words are used to indicate when more, or less, specific detail is required.

Exact
An exact answer is one where numbers are not given in rounded form. The answer will often contain an irrational number such as $\sqrt{3}$, e or π and these numbers should be given in that form when an exact answer is required.

The use of the word 'exact' also tells learners that rigorous (exact) working is expected in the answer to the question.
e.g. Find the exact solution of $\ln x = 2$.
The correct answer is e^2 and not $7.389\ 056$.
e.g. Find the exact solution of $3x = 2$.
The correct answer is $x = \frac{2}{3}$ or $x = 0.\dot{6}$, not $x = 0.67$ or similar.

Prove
Learners are given a statement and must provide a formal mathematical argument which demonstrates its validity. A formal proof requires a high level of mathematical detail, with candidates clearly defining variables, correct algebraic manipulation and a concise conclusion.

Show that
Learners are given a result and have to show that it is true. Because they are given the result, the explanation has to be sufficiently detailed to cover every step of their working.
e.g. Show that the curve $y = x \ln x$ has a stationary point $\left(\frac{1}{e}, -\frac{1}{e}\right)$.

Determine
This command word indicates that justification should be given for any results found, including working where appropriate.

Verify
A clear substitution of the given value to justify the statement is required.

Find, Solve, Calculate
These command words indicate, while working may be necessary to answer the question, no justification is required. A solution could be obtained from the efficient use of a calculator, either graphically or using a numerical method.

Give, State, Write down
These command words indicate that neither working nor justification is required.

2

In this question you must show detailed reasoning.
When a question includes this instruction learners must give a solution which leads to a conclusion showing a detailed and complete analytical method. Their solution should contain sufficient detail to allow the line of their argument to be followed. This is not a restriction on a learner's use of a calculator when tackling the question, e.g. for checking an answer or evaluating a function at a given point, but it is a restriction on what will be accepted as evidence of a complete method.

In these examples variations in the structure of the answers are possible, for example giving the integral as $\ln(x+\sqrt{x^2-16})$ in example 2, and different intermediate steps may be given.

Example 1:
Express $-4+2\mathrm{i}$ in modulus-argument form.

The answer is $\sqrt{20}(\cos 2.68 + \mathrm{i}\sin 2.68)$,
but the learner *must* include the steps
$|-4+2\mathrm{i}| = \sqrt{16+4} = \sqrt{20}$,
$\arg(-4+2\mathrm{i}) = \pi - \tan^{-1}(0.5) = 2.68$. Using a calculator in complex mode to convert to modulus-argument form would not result in a complete analytical method.

Example 2:
Evaluate $\int_4^5 \frac{1}{\sqrt{x^2-16}}\,\mathrm{d}x$.

The answer is $\ln(2)$, but the learner *must* include at least $\lim_{a\to 4}\left[\operatorname{ar\,cosh}\left(\frac{x}{4}\right)\right]_a^5$ and the substitution $\ln\left(\frac{5}{4}+\sqrt{\frac{25}{16}-1}\right) - \ln(1+\sqrt{0})$. Just writing down the answer using the definite integral function on a calculator would therefore not be awarded any marks.

Example 3:
Solve the equation $2x^3 - 11x^2 + 22x - 15 = 0$.

The answer is $1.5,\ 2 \pm \mathrm{i}$, but the learner *must* include steps to find a real root or corresponding factor, find the factor $(2x-3)$ and factorise the cubic then solve the quadratic. Just writing down the three roots by using the cubic equation solver on a calculator would not be awarded any marks.

Hence
When a question uses the word 'hence', it is an indication that the next step should be based on what has gone before. The intention is that learners should start from the indicated statement.
e.g.
You are given that $\mathrm{f}(x) = 2x^3 - x^2 - 7x + 6$.
Show that $(x-1)$ is a factor of $\mathrm{f}(x)$.
Hence find the three factors of $\mathrm{f}(x)$.

Hence or otherwise
This is used when there are multiple ways of answering a given question. Learners starting from the indicated statement may well gain some information about the solution from doing so, and may already be some way towards the answer. The command phrase is used to direct learners towards using a particular piece of information to start from or to a particular method. It also indicates to learners that valid alternate methods exist which will be given full credit, but that they may be more time-consuming or complex.
e.g. Show that $(\cos x + \sin x)^2 = 1 + \sin 2x$ for all x.
Hence, or otherwise, find the derivative of $(\cos x + \sin x)^2$.

You may use the result
When this phrase is used it indicates a given result that learners would not normally be expected to know, but which may be useful in answering the question.

The phrase should be taken as permissive; use of the given result is not required.

Plot
Learners should mark points accurately on graph paper. They will either have been given the points or have had to calculate them. They may also need to join them with a curve or a straight line, or draw a line of best fit through them.
e.g. Plot this additional point on the scatter diagram.

Sketch
Learners should draw a diagram, not necessarily to scale, showing the main features of a curve. These are likely to include at least some of the following.

- Turning points
- Asymptotes
- Intersection with the y-axis
- Intersection with the x-axis
- Behaviour for large x (+ or –)

Any other important features should also be shown.

e.g. Sketch the curve with equation $y = \frac{1}{(x-1)}$

Draw
Learners should draw to an accuracy appropriate to the problem. They are being asked to make a sensible judgement about the level of accuracy which is appropriate.
e.g. Draw a diagram showing the forces acting on the particle.
e.g. Draw a line of best fit for the data.

Other command words
Other command words, for example "explain", will have their ordinary English meaning.

Overarching Themes

These Overarching Themes should be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content in this specification. These statements are intended to direct the teaching and learning of A Level Further Mathematics, and they will be reflected in assessment tasks.

OT1 Mathematical argument, language and proof

	Knowledge/Skill
OT1.1	Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable
OT1.2	Understand and use mathematical language and syntax as set out in the content
OT1.3	Understand and use language and symbols associated with set theory, as set out in the content
OT1.4	Understand and use the definition of a function; domain and range of functions
OT1.5	Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics

2

OT2 Mathematical problem solving

	Knowledge/Skill
OT2.1	Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved
OT2.2	Construct extended arguments to solve problems presented in an unstructured form, including problems in context
OT2.3	Interpret and communicate solutions in the context of the original problem
OT2.4	Not Applicable to A Level Further Mathematics
OT2.5	Not Applicable to A Level Further Mathematics
OT2.6	Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle
OT2.7	Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems

OT3 Mathematical modelling

	Knowledge/Skill
OT3.1	Translate a situation in context into a mathematical model, making simplifying assumptions
OT3.2	Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student)
OT3.3	Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student)
OT3.4	Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate
OT3.5	Understand and use modelling assumptions

2c. Content of Pure Core (Mandatory papers Y540 and Y541)

Introduction to Pure Core.

In **Pure Core** learners will extend and deepen their knowledge of proof, algebra, functions, calculus, vectors and differential equations studied in A Level Mathematics. They will also broaden their knowledge into other areas of pure mathematics that underpin the further study of mathematics and other numerate subjects with complex numbers, matrices, polar coordinates and hyperbolic functions.

4.01 Proof
Proof by induction is introduced, including its application in proofs on sums of simple series, powers of matrices and divisibility.

4.02 Complex Numbers
Complex numbers and their basic arithmetic are introduced, including in modulus-argument form. They are used to solve polynomial equations with real coefficients and to define loci on the Argand diagram. De Moivre's theorem is used to develop trigonometrical relationships. Roots of complex numbers are used to help to solve geometrical problems using an Argand diagram.

4.03 Matrices
Matrix arithmetic is introduced and applied to linear transformations in 2-D, and some in 3-D, including the concept of invariance. Determinants and inverses of 2×2 and 3×3 matrices are found and used to solve matrix equations.

4.04 Further Vectors
Vector equations of lines and planes are studied; methods for finding angles and distances between points, lines and planes are developed. Scalar and vector products are introduced, and used in a variety of geometrical problems. How planes intersect in 3-D space is considered, and matrices are used to find the point(s) of intersection.

4.05 Further Algebra
Relationships between roots of and coefficients of polynomials are explored. Techniques involving partial fractions are developed.

4.06 Series
Simple series are summed, using standard formulae and the method of differences. Partial fractions are used to sum series.

4.07 Hyperbolic functions
Hyperbolic functions are introduced and their inverses are used in integration.

4.08 Further Calculus
Maclaurin series are used to approximate functions. Integration techniques are extended to include improper integrals, volumes of revolution, mean values of functions and partial fractions. Inverse trigonometric functions are defined and used for integration.

4.09 Polar Coordinates Curves defined in polar coordinates are explored, including finding the area enclosed by a curve.

4.10 Differential Equations
The work in A Level Mathematics is extended to include the integrating factor method for first order differential equations. The general 2nd order linear differential equation is solved, including SHM and damped oscillations. Simple systems of linear 1st order differential equations are explored.

Assumed knowledge

Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. All of this content is assumed, but will only be explicitly assessed where it appears in this specification.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners could use graphing software to investigate the relationships between graphical and algebraic representations, including complex numbers, vectors, implicit and parametric curves, and differential equations.

2

2. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including derivatives and integrals, to solve equations, including differential equations, to manipulate matrices and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

3. Visualisation: Learners could use appropriate software to visualise situations in 3-D relating to lines and planes, and to linear transformations.

4. Spreadsheets: Learners could use spreadsheet software to investigate numerical methods, sequences and series, for modelling and to generate tables of values for functions.

Content of Pure Core (Mandatory papers Y540 and Y541)

When this course is being co-taught with OCR's AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR Reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.01 Proof				
4.01a 4.01b	Mathematical induction	a) Be able to construct proofs using mathematical induction. *This topic may be tested using any relevant content including divisibility, powers of matrices and results on powers, exponentials and factorials.* *e.g. Prove that* $\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}^n = \begin{pmatrix}1 & 0\\ n & 1\end{pmatrix}$ *for* $n \in \mathbb{Z}^+$. *Prove that* $7^n - 3^n$ *is divisible by* 4 *for* $n \in \mathbb{Z}^+$. *Prove that* $2^n > 2n$ *for* $n \geq 3, n \in \mathbb{Z}$.	b) Be able to construct proofs of a more demanding nature, including conjecture followed by proof. *This topic may be tested using any relevant content including sums of series.* *e.g. Prove that* $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$ *for all* $n \in \mathbb{Z}$. *Prove that* $3^n > n^3$ *for* $n \geq 4, n \in \mathbb{Z}$. *Prove that* $n! < n^n$ *for* $n \geq 1, n \in \mathbb{Z}$. *Prove that* $(1+x)^n \geq 1 + nx$ *for any real number* $x > -1$ *and* $n \in \mathbb{Z}^+$. *Prove that the* n*th derivative of* $x^2 e^x$ *is* $(x^2 + 2nx + n(n-1))e^x$.	A1

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.02 Complex Numbers				
4.02a	The language of complex numbers	a) Understand the language of complex numbers. *Know the meaning of "real part", "imaginary part", "conjugate", "modulus" and "argument" of a complex number.*		B2 B5 B3
4.02b 4.02d		b) Be able to express a complex number z in either cartesian form $z = x + \mathrm{i}y$, where $\mathrm{i}^2 = -1$, or modulus-argument form $z = r(\cos\theta + \mathrm{i}\sin\theta) = [r, \theta] = r\mathrm{cis}\theta$, where $r \geq 0$ is the modulus of z and θ, measured in radians is the argument of z.	d) Understand and be able to use the exponential form, $r\mathrm{e}^{\mathrm{i}\theta}$, of a complex number.	B9
4.02c		c) Understand and be able to use the notation: z, z^*, $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $\arg(z)$, $\lvert z \rvert$. *Includes knowing that a complex number is zero if and only if both the real and imaginary parts are zero.* *The principal argument of a complex number, for uniqueness, will be taken to lie in either of the intervals* $[0,\ 2\pi)$ *or* $(-\pi, \pi]$. *Learners may use either as appropriate unless the interval is specified.* *In stage 1 knowledge of radians is assumed: see H240 section 1.05d.*		

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.02e	Basic operations	e) Be able to carry out basic arithmetic operations $(+,-,\times,\div)$ on complex numbers in both cartesian and modulus-argument forms. *In stage 1 knowledge of radians and compound angle formulae is assumed: see H240 sections 1.05d and 1.05l.* *Learners may use the results* $z_1 z_2 = [r_1 r_2, \theta_1 + \theta_2]$ *and* $\frac{z_1}{z_2} = \left[\frac{r_1}{r_2}, \theta_1 - \theta_2\right]$.		B2 B5 B6
4.02f		f) Convert between cartesian and modulus-argument forms.		
4.02g	Solution of equations	g) Know that, for a polynomial equation with real coefficients, complex roots occur in conjugate pairs.		B1 B3
4.02h		h) Be able to find algebraically the two square roots of a complex number. *e.g. By squaring and comparing real and imaginary parts.*		
4.02i		i) Be able to solve quadratic equations with real coefficients and complex roots.		
4.02j		j) Be able to use conjugate pairs, and the factor theorem, to solve or factorise cubic or quartic equations with real coefficients. *Where necessary, sufficient information will be given to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics.*		

2

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.02k 4.02m 4.02l	Argand diagrams	k) Be able to use and interpret Argand diagrams. *e.g. To represent and interpret complex numbers geometrically.* *Understand and use the terms "real axis" and "imaginary axis".* l) Understand the geometrical effects of taking the conjugate of a complex number, and adding and subtracting two complex numbers.	m) Understand the geometrical effects of multiplying and dividing two complex numbers. *Includes raising complex numbers to positive integer powers.*	B4 B6
4.02n	Euler's formula		n) Know and be able to use Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$. *e.g. To express, and work with, complex numbers in the forms* $r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\text{cis}\,\theta = [r,\theta]$.	B6 B9

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.02o	Loci	o) Be able to illustrate equations and inequalities involving complex numbers by means of loci in an Argand diagram. *i.e. Circle of the form* $\lvert z-a\rvert = k$ *, half-lines of the form* $\arg(z-a)=b$ *, lines of the form* $\mathrm{Re}(z)=k$ *,* $\mathrm{Im}(z)=k$ *and* $\lvert z-a\rvert = \lvert z-b\rvert$ *, and regions defined by inequalities in these forms.* *To include the convention of dashed and solid lines to show exclusion and inclusion respectively.* *No shading convention will be assumed. If not directed, learners should indicate clearly which regions are included.* *In stage 1 knowledge of radians is assumed: see H240 section 1.05d.*		B7
4.02p		p) Understand and be able to use set notation in the context of loci. *e.g. The region* $\lvert z-a\rvert > k$ *where* $z = x+\mathrm{i}y$ *,* $a = x_a + \mathrm{i}y_a$ *and* $k>0$ *may be represented by the set* $\{x+\mathrm{i}y : (x-x_a)^2+(y-y_a)^2>k^2\}$. *In stage 1 knowledge of radians is assumed: see H240 section 1.05d.*		

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.02q	De Moivre's theorem		q) Understand de Moivre's theorem and use it to find multiple-angle formulae and sums of series involving trigonometric and/or exponential terms. *Express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle.* *e.g.* $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$. *Use expressions for* $\sin\theta$ *and* $\cos\theta$ *in terms of* $e^{i\theta}$ *or equivalent relationships.* *e.g.* $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$. *Express powers of* $\sin\theta$ *and* $\cos\theta$ *in terms of series of trigonometric ratios of multiples of the fundamental angle.* *e.g.* $\sin^5\theta = \frac{1}{16}(10\sin\theta - 5\sin(3\theta) + \sin(5\theta))$.	B8
4.02r	nth roots		r) Be able to find the n distinct nth roots of $re^{i\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon on an Argand diagram. *Answers may be asked for in either cartesian or modulus-argument form.*	B10, B4
4.02s	Roots of unity		s) Be able to use complex roots of unity to solve geometric problems. *e.g. To locate the roots of unity on an Argand diagram or to prove results about sums of roots of unity.*	B11

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.03 Matrices				
4.03a	The language of matrices	a) Understand the language of matrices. *Understand the meaning of "conformable", "equal", "square", "rectangular", "m by n", "determinant", "zero" and "null", "transpose" and "identity" when applied to matrices.* *Learners should be familiar with real matrices and complex matrices.*		C2
4.03b	Matrix addition and multiplication	b) Be able to add, subtract and multiply conformable matrices; multiply a matrix by a scalar. *Learners may perform any operations involving entirely numerical matrices by calculator.* *Includes raising square matrices to positive integer powers.* *Learners should understand the effects on a matrix of adding the zero matrix to it, multiplying it by the zero matrix and multiplying it by the identity matrix.*		C1
4.03c		c) Understand that matrix multiplication is associative but not commutative. *Understand the terms "associative" and "commutative".*		

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...	DfE Ref.
4.03d	Linear transformations	d) Be able to find and use matrices to represent linear transformations in 2-D. *Includes:* • *reflection in either coordinate axis and in the lines* $y = \pm x$ • *rotation about the origin (defined by the angle of rotation* θ*, where the direction of positive rotation is taken to be anticlockwise)* • *enlargement centre the origin (defined by the the scale factor)* • *stretch parallel to either coordinate axis (defined by the invariant axis and scale factor)* • *shear parallel to either coordinate axis (defined by the invariant axis and the image of a transformed point).* *Includes the terms "object" and "image".*		C3
4.03e		e) Be able to find and use matrices to represent successive transformations. *Includes understanding and being able to use the result that the matrix product* $\mathbf{AB}$ *represents the transformation that results from the transformation represented by* $\mathbf{B}$ *followed by the transformation represented by* $\mathbf{A}$.		

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.03f		f) Be able to use matrices to represent single linear transformations in 3-D. *3-D transformations will be confined to reflection in one of the planes $x = 0, y = 0, z = 0$ or rotation about one of the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.* *Includes the terms “plane of reflection” and “axis of rotation”.* *In stage 1 knowledge of 3-D vectors is assumed: see H240 section 1.10b.*		C3
4.03g	Invariance	g) Be able to find invariant points and lines for a linear transformation. *Includes the distinction between invariant lines and lines of invariant points.* [*The 3-D transformations in section 4.03f are excluded.*]		C4
4.03h	Determinants	h) Be able to find the determinant of a 2×2 matrix with and without a calculator. *Use and understand the notation* $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ *or* $\lvert \mathbf{M} \rvert$ *or* $\det \mathbf{M}$.		C5
4.03i		i) Know that the determinant of a 2×2 matrix is the area scale factor of the transformation defined by that matrix, including the effect on the orientation of the image. *Learners should know that a transformation preserves the orientation of the object if the determinant of the matrix which represents it is positive and that the transformation reverses orientation if the determinant is negative, and be able to interpret this geometrically.*		

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.03j 4.03k 4.03l 4.03m		j) Be able to calculate the determinant of a 3×3 matrix with and without a calculator. k) Know that the determinant of a 3×3 matrix is the volume scale factor of the transformation defined by that matrix, including the effect on the preservation of the orientation of the image. *Learners should know that the sign of the determinant determines whether or not the corresponding transformation preserves orientation, but do not need to understand the geometric interpretation of this in 3-D.* l) Understand and be able to use singular and non-singular matrices. *Includes understanding the significance of a zero determinant.* m) Know and be able to use the result that $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$.		C5
4.03n 4.03o 4.03p 4.03q	Inverses	n) Be able to find and use the inverse of a non-singular 2×2 matrix with and without a calculator. o) Be able to find and use the inverse of a non-singular 3×3 matrix with and without a calculator. p) Understand and be able to use simple properties of inverse matrices. *e.g. The result that* $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. q) Understand and be able to use the connection between inverse matrices and inverse transformations.		C6

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.03r 4.03s	Solution of simultaneous equations	r) Be able to solve two or three linear simultaneous equations in two or three variables by the use of an inverse matrix, where a unique solution exists.	s) Be able to determine, for two or three linear simultaneous equations where no unique solution exists, whether the equations have an infinite set of solutions (the equations are consistent) or no solutions (the equations are inconsistent). [*Finding the solution set in the infinite case is excluded.*]	C7
4.03t	Intersection of planes		t) Be able to interpret the solution or failure of solution of three simultaneous linear equations in terms of the geometrical arrangement of three planes. *Learners should know and be able to identify the different ways in which two or three distinct planes can intersect in 3-D space, including cases where two or three of the planes are parallel.* *Learners should understand and be able to apply the geometric significance of the singularity of a matrix in relation to the solution(s) or non-existence of them.* [*Finding the line of intersection of two or more planes is excluded.*]	C8

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.04 Further Vectors				
4.04a	Equation of a straight line	a) Understand and be able to use the equation of a straight line, in 2-D and 3-D, in cartesian and vector form. *Learners should know and be able to use the forms:* $y = mx + c$, $ax + by = c$ *and* $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ *in 2-D, and* $\frac{x-a_1}{u_1} = \frac{y-a_2}{u_2} = \frac{z-a_3}{u_3} (= \lambda)$ *and* $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ *in 3-D.* *Includes being able to convert from one form to another.*		F1
4.04b	Equation of a plane		b) Understand and be able to use the equation of a plane in cartesian and vector form. *Learners should know and be able to use the forms:* $ax + by + cz = d$, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$ *and* $\mathbf{r}.\mathbf{n} = p$. *Includes being able to convert from one form to another.*	F2
4.04c 4.04d	Scalar product	c) Be able to calculate the scalar product and use it both to calculate the angles between vectors and/or lines, and also as a test for perpendicularity. *Includes the notation* $\mathbf{a}.\mathbf{b}$	d) Be able to find the angle between two planes and the angle between a line and a plane.	F3 F4
4.04e 4.04f	Intersections	e) Be able to find, where it exists, the point of intersection between two lines. *Includes determining whether or not lines intersect, are parallel or are skew.*	f) Be able to find the intersection of a line and a plane.	F5

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.04g	Vector product	g) Be able to use the vector product to find a vector perpendicular to two given vectors. *Includes the notation* $\mathbf{a} \times \mathbf{b}$. *When the vector product is required, either a calculator or a formula may be used. The formula below will be given:* $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$. [*The magnitude of the vector product is excluded.*]		Essential content for F2, F3, F5, C6
4.04h 4.04i	Shortest distances		h) Be able to find the distance between two parallel lines and the shortest distance between two skew lines. *For skew lines, the formula* $D = \frac{\lvert(\mathbf{b} - \mathbf{a}).\mathbf{n}\rvert}{\lvert\mathbf{n}\rvert}$*, where* $\mathbf{a}$ *and* $\mathbf{b}$ *are position vectors of points on each line and* $\mathbf{n}$ *is a mutual perpendicular to both lines, will be given.* *Either* $\mathbf{n}$ *will be given, or it must be established from given information including by use of the vector product.* i) Be able to find the shortest distance between a point and a line. *The formula* $D = \frac{\lvert ax_1 + by_1 - c\rvert}{\sqrt{a^2 + b^2}}$ *where the coordinates of the point are* (x_1, y_1) *and the equation of the line is given by* $ax + by = c$*, will be given.*	F5

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.04j			j) Be able to find the shortest distance between a point and a plane. *The formula* $D = \frac{\lvert \mathbf{b.n} - p \rvert}{\lvert \mathbf{n} \rvert}$ *where* $\mathbf{b}$ *is the position vector of the point and the equation of the plane is given by* $\mathbf{r.n} = p$*, will be given.*	F5
4.05 Further Algebra				
4.05a	Roots of equations	a) Understand and be able to use the relationships between the symmetric functions of the roots of polynomial equations and the coefficients. *Up to, and including, quartic equations.* *e.g. For the quadratic equation* $ax^2 + bx + c = 0$ *with roots* α *and* β*,* $\alpha + \beta = -\frac{b}{a}$ *and* $\alpha\beta = \frac{c}{a}$.		D1
4.05b	Transformation of equations	b) Be able to use a substitution to obtain an equation whose roots are related to those of the original equation. *Equations will be of at least cubic degree.*		D2
4.05c	Partial fractions		c) Extend their knowledge of partial fractions up to rational functions in which the denominator may include quadratic factors of the form $ax^2 + c$ for $c > 0$, and in which the degree of the numerator may be equal to, or exceed, the degree of the denominator. *See H240 section 1.02y.*	D4 E4

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.06 Series				
4.06a	Summation of series		a) Understand and be able to use formulae for the sums of integers, squares and cubes and use these to sum related series. *Formulae for* $\sum_{r=1}^{n} r^2$ *and* $\sum_{r=1}^{n} r^3$ *will be given, but learners may be asked to prove them.*	D3
4.06b	Method of differences		b) Understand and be able to use the method of differences to find the sum of a (finite or infinite) series. *Including the use of partial fractions.* *e.g. Find* $\sum_{r=1}^{n} \frac{1}{r(r+2)}$.	D4
4.07 Hyperbolic Functions				
4.07a	Definitions		a) Understand and be able to use the definitions of the hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, in terms of exponentials. *Including the domain and range of each function.*	H1
4.07b			b) Know and be able to sketch the graphs of the hyperbolic functions.	
4.07c			c) Know and be able to use the identity $\cosh^2 x - \sinh^2 x \equiv 1$. *Learners may be asked to derive or use other identities, but no prior knowledge of them is assumed.* [*Prior knowledge of other identities is excluded.*]	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.07d	Differentiation and integration		d) Be able to differentiate and integrate hyperbolic functions.	H2
4.07e 4.07f	Inverse hyperbolic functions		e) Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges. f) Be able to derive and use expressions in terms of logarithms for the inverse hyperbolic functions. *Includes the notation:* $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$ *and* $\text{arsinh}\, x$, $\text{arcosh}\, x$, $\text{artanh}\, x$.	H3 H4
4.08 Further Calculus				
4.08a 4.08b	Maclaurin series		a) Be able to find the Maclaurin series of a function, including the general term. b) Recognise and be able to use the Maclaurin series for e^x, $\sin x$, $\cos x$, $\ln(1+x)$ and $(1+x)^n$, and functions based on these. *The interval of validity should be understood.* [*Proof of the interval of validity and the use of non-real values of* x *are excluded.*]	D5 D6
4.08c	Improper integrals		c) Be able to evaluate improper integrals where either the integrand is undefined at a value in the range of integration or where the range of integration is infinite. *e.g.* $\int_0^2 \frac{1}{\sqrt{x}}\,dx$ *or* $\int_1^\infty \frac{1}{x^2}\,dx$.	E1

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.08d	Volumes of solids of revolution		d) Be able to derive formulae for and calculate volumes of solids of revolution. *To include solids generated using either coordinate axis as the axis of rotation, and the volume of a solid formed by rotation of a region between two curves.* *This includes curves defined parametrically.*	E2
4.08e	Mean values		e) Understand and be able to evaluate the mean value of a function. *Includes the use of: mean value* $= \frac{1}{b-a}\int_a^b \mathrm{f}(x)\,\mathrm{d}x$.	E3
4.08f	Partial fractions		f) Be able to integrate using partial fractions. *See Further Algebra section 4.05c and H240 section 1.02y for permitted forms.*	E4
4.08g	Inverse trigonometric and hyperbolic functions		g) Be able to derive and use the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\sinh^{-1} x$, $\cosh^{-1} x$ and $\tanh^{-1} x$.	E5
4.08h	Further integration		h) Be able to integrate functions of the form: $\frac{1}{\sqrt{a^2-x^2}}$, $\frac{1}{a^2+x^2}$, $\frac{1}{\sqrt{x^2-a^2}}$ and $\frac{1}{\sqrt{x^2+a^2}}$ and use an appropriate inverse trigonometric or hyperbolic substitution for the evaluation of associated definite or indefinite integrals.	E6 H5

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.09 Polar Coordinates				
4.09a	Polar coordinates		a) Understand and be able to use polar coordinates (using the convention $r \geq 0$) and be able to convert between polar and cartesian coordinates.	G1
4.09b	Sketching curves		b) Be able to sketch polar curves, with r given as a function of θ. *Identify significant features of polar curves such as symmetry, and least and greatest values of r.* *Includes use of trigonometric functions.*	G2
4.09c	Area		c) Be able to find the area enclosed by a polar curve. *Be able to use the formula $\frac{1}{2}\int r^2\,\mathrm{d}\theta$.*	G3

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.10 Differential Equations				
4.10a	General and particular solutions		a) Understand the difference between, and be able to find, general and particular solutions to differential equations. *Includes understanding that the general solution will include arbitrary constant(s) and that the particular solution may be found from initial or boundary conditions.*	I2
4.10b	Modelling		b) Be able to use differential equations in modelling in kinematics and in other contexts. *Includes use of Newton's second law of motion and the language of kinematics including the notation* $v = \dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$ *and* $a = \dot{v} = \ddot{x} = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$. *Includes problems involving variable force.* *Problems may include formulating differential equations which leaners cannot solve analytically.* [*Problems involving either variable mass or the form* $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ *are excluded.*]	I3, I2
4.10c	Integrating factor method for first order differential equations		c) Be able to find and use an integrating factor $\mathrm{e}^{\int \mathrm{P}(x)\mathrm{d}x}$ to solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$. *Includes recognising when it is appropriate to do so, rearranging into the given form when necessary.*	I1

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.10d	Second order homogeneous differential equations		d) Be able to solve differential equations of the form $y'' + ay' + by = 0$, where a and b are constants, by using the auxiliary equation. *Includes rearranging into the given form when necessary.* *Learners should be able to interpret the sign of the discriminant of the auxiliary equation and how it determines the form of the complementary function.* *Including the cases when the roots of the auxiliary equation are:* *(i) distinct and real,* *(ii) repeated,* *(iii) complex.*	I4 I6 I2
4.10e	Second order non-homogeneous differential equations		e) Be able to solve differential equations of the form $y'' + ay' + by = \mathrm{f}(x)$, where a and b are constants, by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $\mathrm{f}(x)$ is a polynomial, exponential or trigonometric function). *Includes cases where the form of the complementary function affects the choice of trial integral for the particular integral.* *Includes cases where the form of the particular integral is given.*	I5

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…	DfE Ref.
4.10f	Simple harmonic motion		f) Be able to solve the equation for simple harmonic motion (SHM) $\ddot{x} = -\omega^2 x$ and relate the solution to the motion. *Includes use of the formulae* $x = A\cos(\omega t) + B\sin(\omega t)$ *and* $x = R\sin(\omega t + \varphi)$ *in* *modelling situations.* *Learners may quote these formulae without proof when not asked to derive it or to solve the SHM equation.*	I7
4.10g	Damped oscillations		g) Be able to model damped oscillations using second order differential equations and interpret their solutions. *The terms "underdamping", "overdamping" and "critical damping" should be known and understood informally.*	I8
4.10h	Linear systems		h) Be able to analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled, simultaneous, first order differential equations, and be able to solve them. *e.g. Predator-prey models, continuous population models, industrial processes.* *Includes solution by eliminating one variable to produce a single second order differential equation.* *Systems will be of the form* $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by + \mathrm{f}(t), \frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy + \mathrm{g}(t)$ *or easily reducible to this form.*	I9

2d. Content of Statistics (Optional paper Y542)

Introduction to Statistics.

In **Statistics** learners will explore the theory which underlies the statistics content in A Level Mathematics, as well as extending their tool box of statistical concepts and techniques. This area covers probability involving combinatorics, probability distributions for discrete and continuous random variables, hypothesis tests and confidence intervals for a population mean, chi-squared tests, non-parametric tests, correlation and regression.

5.01 Probability
The work on probability in A Level Mathematics is extended to include problems involving arrangements and selections.

5.02 Discrete Random Variables
The general concept of a discrete random variable introduced in A Level Mathematics is further developed, along with the calculation of expectation and variance. The discrete uniform, binomial, geometric and Poisson distributions are studied.

5.03 Continuous Random Variables
The general concept of a continuous random variable introduced in A Level Mathematics is further developed, including probability density functions and cumulative distribution functions. Calculus is used to find expectation, median and quartiles.

5.04 Linear Combinations of Random Variables
Formulae are introduced and applied for linear combinations.

5.05 Hypothesis Tests and Confidence Intervals
The study of hypothesis tests is formalised and developed further, including the central limit theorem. The concept of a confidence interval is introduced, applied in context.

5.06 Chi-squared Tests
The use of a Chi-squared test to test for independence and goodness of fit is explored, including the interpretation of the results.

5.07 Non-parametric Tests
The concept of a non-parametric test is introduced and explored through the Wilcoxon signed-rank tests and the Wilcoxon rank-sum test (or Mann-Whitney U test), and applied in hypothesis tests concerning population median and identity of populations.

5.08 Correlation
The concept of correlation introduced in A Level Mathematics is formalised and explored further, including the study of rank correlation.

5.09 Linear Regression
Regressions lines are calculated and used in context for estimation.

Assumed knowledge
Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. They are also assumed to know the content of Pure Core (Y540 and Y541). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Learners should use spreadsheets or statistical software to generate tables and diagrams, and to perform standard statistical calculations.

2. Hypothesis tests: Learners should use spreadsheets or statistical software to carry out hypothesis tests using the techniques in this paper.

3. Probability: Learners should use random number generators, including spreadsheets, to simulate tossing coins, rolling dice etc, and to investigate probability distributions.

4 Central limit theorem: Learners should use simulations to investigate the central limit theorem, including sampling from a variety of distributions.

5 Use of data: Learners are expected to have explored different data sets, using appropriate technology, during the course. No particular data set is expected to be studied, and there will not be any pre-release data.

Hypothesis Tests

Hypotheses should be stated in terms of parameter values (where relevant) and the meanings of symbols should be stated. For example,
" $H_0: p = 0.7, H_1: p < 0.7$, where p is the population proportion in favour of the resolution".

Conclusions should be stated in such a way as to reflect the fact that they are not certain. For example,
"There is evidence at the 5% level to reject H_0. It is likely that the mean mass is less than 500 g."
"There is no evidence at the 2% level to reject H_0. There is no reason to suppose that the mean journey time has changed."

Some examples of incorrect conclusion are as follows:
" H_0 is rejected. Waiting times have increased."
"Accept H_0. Plants in this area have the same height as plants in other areas."

Content of Statistics (Optional paper Y542)

When this course is being co-taught with AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.01 Probability			
5.01a	Probability	a) Be able to evaluate probabilities by calculation using permutations and combinations. *Includes the terms "permutation" and "combination".* *Includes the notation* ${}_n\mathrm{P}_r = {}^n\mathrm{P}_r$ *and* ${}_n\mathrm{C}_r = {}^n\mathrm{C}_r$. *For underlying content on probability see H240 section 2.03.*	
5.01b		b) Be able to evaluate probabilities by calculation in contexts involving selections and arrangements. *Selection problems include, for example, finding the probability that 3 vowels and 2 consonants are chosen when 5 letters are chosen at random from the word 'CALCULATOR'.* *Arrangement problems only involve arrangement of objects in a line and include:* *1. repetition, e.g. the probability that the word 'ARTIST' is formed when the letters of the word 'STRAIT' are chosen at random.* *2. restriction, e.g. the probability that two consonants are (or are not) next to each other when the letters of the word 'TRAITS' are placed in a random order.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.02 Discrete Random Variables			
5.02a	Probability distributions for general discrete random variables	a) Understand and be able to use discrete probability distributions. *Includes using and constructing probability distribution tables and functions relating to a given situation involving a discrete random variable.* *Any defined non-standard distribution will be finite.*	
5.02b		b) Understand and be able to calculate the expectation and variance of a discrete random variable. *Includes knowing and being able to use the formulae* $\mu = \mathrm{E}(X) = \sum x_i p_i$ $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 \, p_i = \sum x_i^2 p_i - \mu^2.$ [*Proof of these results is excluded.*]	
5.02c		c) Know and be able to use the effects of linear coding on the mean and variance of a random variable.	
5.02d	The binomial distribution	d) Know and be able to use the formulae $\mu = np$ and $\sigma^2 = np(1-p)$ for a binomial distribution. *[Proof of these results is excluded.]* *For the underlying content on binomial distributions, see H240 sections 2.04b and 2.04c.*	
5.02e	The discrete uniform distribution	e) Know and be able to use the conditions under which a random variable will have a discrete uniform distribution, and be able to calculate probabilities and the mean and variance for a given discrete uniform distribution. *Includes use of the notation* $X \sim \mathrm{U}(n)$ *for the uniform distribution over the interval* $[1, n]$.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.02f	The geometric distribution	f) Know and be able to use the conditions under which a random variable will have a geometric distribution. *Includes use of the notation* $X \sim \text{Geo}(p)$*, where* X *is the number of trials up to and including the first success.*	
5.02g		g) Be able to calculate probabilities using the geometric distribution. *Learners may use the formulae* $\text{P}(X = x) = (1-p)^{x-1}p$ *and* $\text{P}(X > x) = (1-p)^x$.	
5.02h		h) Know and be able to use the formulae $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{1-p}{p^2}$ for a geometric distribution. [*Proof of these results is excluded.*]	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.02i	The Poisson distribution	i) Understand informally the relevance of the Poisson distribution to the distribution of random events, and be able to use the Poisson distribution as a model. *Includes use of the notation* $X \sim \mathrm{Po}(\lambda)$, *where* X *is the number of events in a given interval.*	
5.02j		j) Understand and be able to use the formula . $\mathrm{P}(X = x) = e^{-\lambda}\dfrac{\lambda^x}{x!}$	
5.02k		k) Be able to calculate probabilities using the Poisson distribution, using appropriate calculator functions. *Learners are expected to have a calculator with the ability to access probabilities from the Poisson distribution.* [*Use of the Poisson distribution to calculate numerical approximations for a binomial distribution is excluded.*]	
5.02l		l) Know and be able to use the conditions under which a random variable will have a Poisson distribution. *Learners will be expected to identify which of the modelling conditions [assumptions] is/are relevant to a given scenario and to explain them in context.*	
5.02m		m) Be able to use the result that if $X \sim \mathrm{Po}(\lambda)$ then the mean and variance of X are each equal to λ.	
5.02n		n) Know and be able to use the result that the sum of independent Poisson variables has a Poisson distribution.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.03 Continuous Random Variables			
5.03a	Continuous random variables		a) Understand and be able to use the concept of a continuous random variable, a probability density function (p.d.f.) and a cumulative distribution function (c.d.f). *Includes the normal, continuous uniform and exponential distributions.* *Includes understanding informally the link between the exponential and Poisson distributions.* *Includes knowing and being able to use the formula for the mean and variance of the continuous uniform and exponential distributions.* *For the underlying content on normal distributions, see H240 sections 2.04e, 2.04f and 2.04g.*
5.03b	Probability density functions		b) Be able to use a probability density function (including where defined piecewise) to solve problems involving probabilities. *Includes knowing and being able to use* $\int_{-\infty}^{\infty} \mathrm{f}(x)\,\mathrm{d}x = 1$.
5.03c			c) Be able to calculate the mean and/or variance of a distribution using the formulae $\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$ and $\sigma^2 = \mathrm{Var}(X) = \int (x-\mu)^2\mathrm{f}(x)\,\mathrm{d}x = \int x^2\mathrm{f}(x)\,\mathrm{d}x - \mu^2$.
5.03d			d) Be able to use the general result $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x)\mathrm{f}(x)\,\mathrm{d}x$, where $\mathrm{f}(x)$ is the probability density function of the continuous random variable X and $\mathrm{g}(x)$ is a function of X.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.03e	Cumulative distribution functions		e) Be able to find and use a cumulative distribution function (including where defined piecewise) to solve problems involving probabilities. *Includes being able to use* $\mathrm{F}(x) = \mathrm{P}(X \le x) = \int_{-\infty}^{x} \mathrm{f}(t)\,\mathrm{d}t$
5.03f			f) Know and be able to use the relationship between the probability density function, $\mathrm{f}(x)$, and the cumulative distribution function, $\mathrm{F}(x)$, and use either to evaluate the median, quartiles and other percentiles.
5.03g			g) Be able to find and use the cumulative distribution functions of related variables. *e.g. Given the c.d.f. of X, find the c.d.f. of Y and hence the p.d.f. of Y where $Y = X^3$.*
5.04 Linear Combinations of Random Variables			
5.04a	Linear combinations of any random variables		a) Be able to use the following results, including the cases where $a = b = \pm 1$ and/or $c = 0$: 1. $\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$, 2. if X and Y are independent then $\mathrm{Var}(aX + bY + c) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y)$.
5.04b	Linear combinations of normal random variables		b) Be able to use the following results: 1. if X has a normal distribution then $aX + b$ has a normal distribution, 2. if X and Y have independent normal distributions then $aX + bY$ has a normal distribution.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.05 Hypothesis Tests and Confidence Intervals			
5.05a	The distribution of $\bar{X}$ and the central limit theorem		a) Know that for any randomly and independently selected sample, X_i, of size n taken from a population, then for the sample mean $\bar{X}$: 1. $\mathrm{E}(\bar{X}) = \mu$, 2. $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$ and 3. $\bar{X}$ is approximately normally distributed when n is large (approximately $n > 25$). [*Proof of these results is excluded.*]
5.05b	Unbiased estimates of population mean and variance		b) Know that unbiased estimates of the population mean and variance are given by $\frac{\Sigma x}{n}$ and $\frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2\right)$ respectively. [*Proof of these results is excluded.*] *Only an informal understanding of "unbiased" is required.*
5.05c	Using the normal distribution in hypothesis tests		c) Be able to use a normal distribution to carry out a hypothesis test for a population mean in the following cases: 1. a sample drawn from a normal population of known, given or assumed variance, 2. a large sample drawn from any population with known, given or assumed variance, 3. a large sample, drawn from any population with unknown variance.
5.05d	Confidence intervals		d) Be able to use a normal distribution to find a confidence interval for a population mean in each of the above cases.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.06 Chi-squared Tests			
5.06a	Contingency tables	a) Be able to use a chi-squared (χ^2) test with the appropriate number of degrees of freedom to test for independence in a contingency table and interpret the results of such a test. *Rows or columns, as appropriate, should be combined so that each expected frequency is at least 5, and Yates' correction should be used in the special case of a 2×2 table.* *A table of critical values of the χ^2 distribution will be provided.* *Includes calculation of expected frequencies and contributions to the test statistic.* *Questions may require candidates to calculate some expected frequencies and contributions to the test statistic, but will not involve lengthy calculations.*	
5.06b 5.06c	Fitting a theoretical distribution	b) Be able to fit a theoretical distribution, as prescribed by a given hypothesis involving a given ratio, proportion or discrete uniform distribution, to given data. *Questions may require candidates to calculate some expected frequencies, but will not involve lengthy calculations.*	c) Extend their knowledge of fitting distributions to other known or given discrete and continuous distributions. *Questions may require candidates to calculate some expected frequencies, but will not involve lengthy calculations.*
5.06d	Goodness of fit test	d) Be able to use a χ^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit test. *Where necessary, adjacent classes should be combined so that each expected frequency is at least 5.* *A table of critical values of the χ^2 distribution will be provided.*	

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
5.07 Non-parametric tests			
5.07a	Non-parametric tests		a) Understand what is meant by a non-parametric hypothesis test, appreciate situations where such tests are useful and be able to select an appropriate test.
5.07b	The basis of non-parametric tests		b) Understand the basis of sign tests, the Wilcoxon signed-rank test and the Wilcoxon rank-sum test (also known as the Mann-Whitney U test). *Tables of critical values of* T *and* W *will be provided. Learners should know the notation* W_+ *and* W_-.
5.07c	Single-sample hypothesis tests		c) Be able to test a hypothesis concerning a population median using a single-sample sign test and a single-sample Wilcoxon signed-rank test. [*Problems in which observations coincide with the hypothetical population median are excluded.*]
5.07d 5.07e	Paired-sample and two-sample hypothesis tests		d) Understand the difference between a paired-sample test and a two-sample test, and be able to select the appropriate form when solving problems. e) Be able to test for medians or identity of population as appropriate, using a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and (for unpaired samples) a Wilcoxon rank-sum test. [*Problems involving tied ranks are excluded.*]

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.07f	Normal approximations		f) Be able to carry out tests using the Wilcoxon signed-rank test and the Wilcoxon rank-sum test for large samples using the approximations: Wilcoxon signed-rank test $T \sim \mathrm{N}\left(\frac{1}{4}n(n+1), \frac{1}{24}n(n+1)(2n+1)\right)$ Wilcoxon rank-sum test (samples of sizes m and n, with $m \leq n$) $W \sim \mathrm{N}\left(\frac{1}{2}m(m+n+1), \frac{1}{12}mn(m+n+1)\right)$. *Includes the use of continuity corrections.*
5.08 Correlation			
5.08a	Pearson's product-moment correlation coefficient	a) Be able to calculate the product-moment correlation coefficient (pmcc) for a set of bivariate data; raw data or summarised data may be given. *Use of appropriate calculator functions is expected.* *Learners will not be required to enter large amounts of data into a calculator during the examination.*	
5.08b		b) Understand that the value of a correlation coefficient is unaffected by linear coding of the variables.	
5.08c		c) Understand Pearson's product-moment correlation coefficient as a measure of how close data points lie to a straight line.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.08d	Hypothesis tests using Pearson's product-moment correlation coefficient	d) Use and be able to interpret Pearson's product-moment correlation coefficient in hypothesis tests, using either a given critical value, or a p-value and a table of critical values. *When using Pearson's coefficient in a hypothesis test, the data may be assumed to come from a bivariate normal distribution.* *A table of critical values of Pearson's coefficient will be provided.*	
5.08e	Spearman's rank correlation coefficient	e) Be able to calculate Spearman's rank correlation coefficient for a maximum of 10 pairs of data values or ranks. *Includes being able to draw basic conclusions about the meaning of a value of the coefficient in relation to the ranks before, or without, carrying out a hypothesis test.* *Includes understanding the conditions under which the use of rank correlation may be appropriate.* [*Tied ranks are excluded.*]	
5.08f	Hypothesis tests using Spearman's coefficient	f) Be able to carry out a hypothesis test for association in a population. *Includes understanding that this is a non-parametric test, as it makes no assumptions about the population.* *Tables of critical values of Spearman's coefficient will be provided.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
5.08g	Comparison of coefficients	g) Be able to choose between Pearson's product-moment correlation coefficient and Spearman's rank correlation coefficient for a given context. *Includes interpreting a scatter diagram and distinguishing between linear correlation and association.*	
5.09 Linear Regression			
5.09a	Dependent and independent variables	a) Understand the difference between an independent (or controlled) variable and a dependent (or response) variable. *Includes appreciating that, in a given situation, neither parameter may be independent.*	
5.09b 5.09c 5.09d	Calculation of the equation of the regression line	b) Understand the concepts of least squares and regression lines in the context of a scatter diagram. c) Be able to calculate, both from raw data and from summarised data, the equation of the regression line of y on x, where the independent variable (if any) is x. [*The regression line of x on y is excluded in the case when x is independent.*] d) Understand the effect on a regression line of linear coding on one or both variables.	
5.09e	Use of the regression line	e) Be able to use, in the context of a problem, the regression line of y on x to estimate a value of y, and be able to interpret in context the uncertainties of such an estimate.	

2e. Content of Mechanics (Optional paper Y543)

Introduction to Mechanics.

In **Mechanics** learners extend their knowledge of particles, kinematics and forces from A Level Mathematics, using their extended pure mathematical knowledge to explore more complex physical systems. The area covers dimensional analysis, work, energy, power, impulse, momentum, centres of mass, circular motion and variable force.

6.01 Dimensional Analysis
The relationships between physical quantities are analysed by considering their dimensions (length, mass and time), in order to construct or check models.

6.02 Work, Energy and Power
The fundamental concepts of work, energy and power are introduced, including kinetic energy, gravitational potential energy and elastic potential energy. The principle of conservation of mechanical energy is used to solve problems.

6.03 Impulse and Momentum
Problems involving collisions in 1-D and 2-D are studied, using the principal of conservation of linear momentum and Newton's experimental law.

6.04 Centre of Mass
Rigid bodies are modelled as particles at their centres of mass. Techniques for finding the centre of mass of a body or system of bodies are explored, including integration.

6.05 Motion in a circle
The motion of a particle in a horizontal or vertical circle is explored, including motion which is not restricted to a circular path.

6.06 Further Dynamics and Kinematics
The techniques of differential equations studied in Pure Core are extended to linear motion under a variable force.

Assumed knowledge
Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. They are also assumed to know the content of Pure Core (Y540 and Y541). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

Resolving forces

The technique of **resolving forces** is found in 'Stage 2' of the A Level mathematics content, and therefore 'Stage 1' learners may not have learned this technique yet. As it is a vital underlying skill in the more advanced mechanics topics met in this paper, it is taken as assumed knowledge for 'Stage 1'. This includes both being able to express a force as two mutually perpendicular components, and being able to find the resultant of two or more forces acting at a point. See sections 6.02b, 6.02l and 6.05c.

Use of technology
To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Spreadsheets: Learners should use spreadsheets to generate tables of values for functions and to investigate functions numerically.

2. Learners should use graphing software for modelling, including kinematics and projectiles, and in visualising physical systems.

3. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including derivatives and integrals, and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

4. Complex problem solving: Learners could use CAS to perform computation when solving complex problems in mechanics, including those which lead to equations or systems that they cannot solve analytically.

5. Practical mechanics: Learners could use computers and/or mobile phones to enrich practical mechanics tasks, using them for data logging, to create videos of moving objects, or to share and analyse data.

Content of Mechanics (Optional paper Y543)

When this course is being co-taught with AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.01 Dimensional Analysis			
6.01a	Dimensional analysis	a) Be able to find the dimensions of a quantity in terms of M, L and T, and understand that some quantities are dimensionless. *Includes understanding and using the notation* $[d]$ *for the dimension of the quantity* d. *Learners are expected to know or be able to derive the dimensions of any quantity for which they know the units. Dimensions of other quantities will be given, or their derivation will be the focus of assessment.*	
6.01b		b) Understand and be able to use the relationship between the units of a quantity and its dimensions.	
6.01c		c) Be able to use dimensional analysis as an error check. *e.g. Verify the relationship that power is proportional to the product of the driving force and the velocity.*	
6.01d		d) Be able to use dimensional analysis to determine unknown indices in a proposed formulation. *e.g. Determine the period of oscillation of a simple pendulum in terms of its length, mass and the acceleration due to gravity,* g.	
6.01e		e) Be able to formulate models and derive equations of motion using a dimensional argument.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.02 Work, Energy and Power			
6.02a 6.02c 6.02b	Work	a) Understand the concept of work done by a force. b) Be able to calculate the work done by a constant force. *The force may not act in the direction of motion of the body and so learners will be expected to resolve forces in two dimensions.*	c) Be able to calculate the work done by a constant force in two dimensions using vectors $(\mathbf{F.x})$ or by a variable force $\left(\int F\mathrm{d}x\right)$ in one dimension only.
6.02d 6.02f 6.02e	Energy	d) Understand the concept of the mechanical energy of a body. *i.e. The kinetic and potential energy.* e) Be able to calculate the gravitational potential energy (mgh) and kinetic energy $\left(\frac{1}{2}mv^2\right)$ of a body.	f) Be able to calculate the kinetic energy of a body using the scalar product $\frac{1}{2}m\mathbf{v.v}$ *Learners may be expected to use the formula* $\mathbf{v.v} = \mathbf{u.u} + 2\mathbf{a.x}$, *in solving a variety of problems, for example in calculating the kinetic energy of a body.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.02g 6.02h	Hooke's law		g) Understand and be able to use Hooke's law, in the form $T = \frac{\lambda x}{l}$, for elastic strings and springs. *Includes an informal understanding of when Hooke's law does not apply.* h) Be able to calculate the elastic potential energy $\left(E = \frac{\lambda x^2}{2l}\right)$ stored in a string or spring. *Learners will be expected to state the formula for the elastic potential energy stored in a string or spring unless they are explicitly asked to derive it.*
6.02i 6.02j	Conservation of energy	i) Understand and be able to use the principle of the conservation of mechanical energy and the work-energy principle for dynamic systems, including consideration of energy loss.	j) Extend their knowledge of the principle of the conservation of mechanical energy and the work-energy principle to systems which include elastic strings or springs.
6.02k 6.02l 6.02m	Power	k) Understand and be able to use the definition of power (the rate at which a force does work). *Includes* $\text{average power} = \frac{\text{work done}}{\text{time elapsed}}$. l) Be able to use the relationship between power, the tractive force and velocity $(P = Fv)$ to solve problems. *e.g. Motion on an inclined plane.* *Includes maximum velocity and speed.* *Learners will be required to resolve forces in two dimensions.*	m) Be able to calculate the power associated with a variable force in two dimensions using the scalar product $P = \mathbf{F.v}$

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.03 Impulse and Momentum			
6.03a 6.03c	Linear momentum	a) Recall and be able to use the definition of linear momentum in one dimension.	c) Recall and be able to use the definition of momentum in two dimensions including the vector form $m\mathbf{v}$.
6.03b 6.03d		b) Understand and be able to apply the principle of conservation of linear momentum in one dimension applied to two particles. *Includes using the formula* $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.	d) Understand and be able to apply the principle of conservation of linear momentum in two dimensions applied to two particles. *Includes using the vector form* $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$.
6.03e	Impulse	e) Understand and be able to use the concept of the impulse imparted by a force.	
6.03f 6.03g		f) Be able to use the relationship between the instantaneous impulse of a force and the change in momentum $(I = mv - mu)$. *The instantaneous impulse is the impulse associated with an instantaneous change in velocity.* *Learners will only be required to apply this to instantaneous events in one dimension.* *e.g.* *The direct impact of two smooth spheres.* *An impulsive force acting in the direction of an inelastic string.* *Questions involving collision(s) between particles may include multiple collisions and the conditions under which further collisions occur.*	g) Understand and be able to apply the impulse – momentum principle in two dimensions including the vector form $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$. *e.g.* *The oblique impact of two smooth spheres.* *A smooth sphere with a fixed plane surface.* *An impulsive force acting at an angle to an inelastic string.*
6.03h			h) Understand and be able to apply the impulse – momentum principle for a constant force expressed as force $\times$ time or for a variable force in one dimension only as $\int F\,\mathrm{d}t$.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.03i	Restitution	i) Recall and be able to use the definition of the coefficient of restitution, including $0 \leq e \leq 1$. [*Superelastic collisions are excluded.*]	
6.03j		j) Understand and be able to use the terms "perfectly elastic" $(e = 1)$ and "inelastic" $(e = 0)$ for describing collisions. *Learners should know that for perfectly elastic collisions there will be no loss of kinetic energy and for inelastic collisions the bodies coalesce and there is maximum loss of kinetic energy.*	
6.03k 6.03l		k) Recall and be able to use Newton's experimental law in one dimension for problems of direct impact. e.g. *Between two smooth spheres* $(v_1 - v_2 = -e(u_1 - u_2))$ *and a smooth sphere with a fixed plane surface* $(v = -eu)$*, where* u *and* v *are the velocities before and after impact.*	l) Extend their knowledge to problems involving Newton's experimental law in two dimensions. *e.g. The oblique impacts of two smooth spheres and a smooth sphere with a fixed plane surface.* *Questions may involve the velocity expressed as a two dimensional vector.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.04 Centre of Mass			
6.04a	Centre of mass		a) Understand and be able to apply the principle that the effect of gravity is equivalent to a single force acting at the body’s centre of mass. *Includes understanding that, in terms of linear motion, a rigid body may be modelled by a particle of the same mass at its centre of mass.*
6.04b			b) Be able to find the position of the centre of mass of a uniform rigid body using symmetry, for example a rectangular lamina.
6.04c			c) Be able to determine the centre of mass of a system of particles or the centre of mass of a composite rigid body. *Questions may involve any of the rigid bodies listed in the Formulae Booklet, but will be limited to compound shapes such as a uniform L-shaped lamina or a hemisphere abutting a cylinder with a common axis.* *Includes composition by addition or subtraction, for example a rectangular lamina with a semicircle attached to one side, or a rectangular lamina with a semicircle removed.*
6.04d			d) Be able to use integration to determine the position of the centre of mass of a uniform lamina or a uniform solid of revolution.
6.04e	Rigid bodies		e) Be able to solve problems involving the equilibrium of a single rigid body under the action of coplanar forces. *e.g. Suspension of a rigid body from a given point or problems involving the toppling or sliding of a rigid body placed on an inclined plane*. *May include rigid bodies which are hinged to a surface.* [*Hinged bodies are excluded.*]

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.05 Motion in a Circle			
6.05a	Uniform motion in a circle	a) Understand and be able to use the definitions of angular velocity, velocity, speed and acceleration in relation to a particle moving in a circular path, or a point rotating in a circle, with constant speed. *Includes the use of both ω and $\dot{\theta}$.*	
6.05b		b) Be able to use and apply the relationships $v = r\dot{\theta}$ and $a = \frac{v^2}{r} = r\dot{\theta}^2 = v\dot{\theta}$ for motion in a circle with constant speed.	
6.05c		c) Be able to solve problems regarding motion in a horizontal circle. *e.g.* *Motion of a conical pendulum.* *Motion on a banked track.* *Problems will be restricted to those involving constant forces but learners will be required to resolve forces in two dimensions.*	
6.05d 6.05e	Motion in a vertical circle	d) Understand the motion of a particle in a circle with variable speed. *In 'Stage 1' Learners will be expected to use energy considerations to calculate the speed of a particle at a given point on a circular path.*	e) Extend their understanding of the motion of a particle in a circle with variable speed to include the radial and tangential components of the acceleration.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
6.05f			f) Be able to solve problems involving motion round a vertical circle including motion which is not restricted to a circular path. *This is restricted to a combination of motion in a circle and free fall.* *e.g. The subsequent motion of a particle moving on the outside of a smooth circular surface.* *The motion of a particle on a string moving in a vertical circle and then as a projectile.*
6.06 Further Dynamics and Kinematics			
6.06a	Linear motion under a variable force		a) Be able to use $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ or $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$ to model the linear motion of a particle under the action of a variable force in one dimension only. *Learners will be required to solve problems in which the corresponding differential equation can be solved by either the method of separation of variables or an integrating factor.*

2f. Content of Discrete Mathematics (Optional paper Y544)

Introduction to Discrete Mathematics.

Discrete Mathematics is the part of mathematics dedicated to the study of discrete objects. Learners will study both pure mathematical structures and techniques and their application to solving real-world problems of existence, construction, enumeration and optimisation. Areas studied include counting, graphs and networks, algorithms, critical path analysis, linear programming and game theory.

7.01 Mathematical Preliminaries
Learners are introduced to the fundamental categorisation of problems as existence, construction, enumeration and optimisation. They are also introduced to counting techniques that have a wide application across Discrete Mathematics.

7.02 Graphs and Networks
Graphs and networks are introduced as mathematical objects that can be used to model real world systems involving connections and relationships. The pure mathematics of graph theory is studied including isomorphism, and Eulerian, Hamiltonian and planar graphs.

7.03 Algorithms
The algorithmic approach to problem solving in introduced via sorting and packing problems. The run-time and order of an algorithm are studied, including the hierarchy of orders.

7.04 Network Algorithms
Problems involving networks are introduced: shortest path; minimum connector; travelling salesperson and route inspection.
Standard network algorithms are studied and used to solve these problems.

7.05 Decision Making in Project Management
Networks are applied to decision making, in particular to activity networks and critical path analysis, including scheduling.

7.06 Graphical Linear Programming
The concept of linear programming is explored as a tool for optimisation, including integer programming. Linear programmes are solved graphically.

7.07 The Simplex Algorithm
The simplex algorithm is used to solve optimisation problems.

7.08 Game Theory
Problems of conflict and cooperation are explored using game theory, including both pure and mixed strategies.

Assumed knowledge
Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. They are also assumed to know the content of Pure Core (Y540 and Y541). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

The use of algorithms

Learners will only be expected to use specific algorithms if instructed to do so in the question. For example a list may be sorted by inspection unless a question specifically asks for the use of a sorting algorithm, and lengths of shortest paths may be found by inspection unless a question specifically asks for the use of Dijkstra's algorithm.

The Formulae Booklet contains sketches of some of the algorithms found in this area. The focus of the study of algorithms in this area should be on understanding the theory and processes, not on memorisation or practising the rote application of algorithms by hand. In the classroom, technology should be used to demonstrate the power of these algorithms in large scale problems; in the assessment, learners will be asked to demonstrate their understanding of the application of algorithms to small scale problems only.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners could use graphing software to perform graphical linear programming, and to investigate the effects on the solution of changing coefficients and parameters.

2. Networks and network algorithms: Learners could use software to investigate networks and to implement network algorithms, in particular for networks which are too large to work with by hand.

3. Algorithms: Learners could use spreadsheets or a suitable programming language to implement simple algorithms and to understand how to create algorithms to perform simple tasks.

4. Computer Algebra System (CAS): Learners could use CAS software to draw and manipulate graphs, to explore algebraic relationships. This is best done in conjunction with other software such as graphing tools and spreadsheets.

5. Simulation: Learners could use spreadsheets to simulate contexts in game theory, including investigating the long term effects of particular strategies.

Content of Discrete Mathematics (Optional paper Y544)

When this course is being co-taught with AS Level Further Mathematics A (H235) the ‘Stage 1’ column indicates the common content between the two specifications and the ‘Stage 2’ column indicates content which is particular to this specification. In each section the OCR reference lists ‘Stage 1’ statements before ‘Stage 2‘ statements.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.01 Mathematical Preliminaries			
7.01a	Types of problem	a) Understand and be able to use the terms “existence’” “construction”, “enumeration” and “optimisation” in the context of problem solving. *Includes classifying a given problem into one or more of these categories.*	
7.01b	Set notation	b) Understand and be able to use the basic language and notation of sets. *Includes the term “partition” and counting the number of partitions of a set including with constraints.*	
7.01c	The pigeonhole principle	c) Be able to use the pigeonhole principle in solving problems.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.01d	Arrangement and selection problems	d) Understand and use the multiplicative principle. Includes knowing that the number of arrangements of n distinct objects is $\prod_{r=1}^{n} r = n!$.	
7.01e		e) Be able to enumerate the number of ways of obtaining an ordered subset (permutation) of r elements from a set of n distinct elements. *Includes using the notation* ${}_{n}\mathrm{P}_{r} = {}^{n}\mathrm{P}_{r}$.	
7.01f		f) Be able to enumerate the number of ways of obtaining an unordered subset (combination) of r elements from a set of n distinct elements. *Includes using the notation* ${}_{n}\mathrm{C}_{r} = {}^{n}\mathrm{C}_{r}$.	
7.01g 7.01h		g) Be able to solve problems about enumerating the number of arrangements of objects in a line, including those involving: 1. repetition, *e.g. how many different eight digit numbers can be made from the digits of* 12333210*?* 2. restriction, *e.g. how many different eight digit numbers can be made from the digits of* 12333210 *if the two 2s cannot be next to each other?*	h) Be able to solve problems about enumerating the number of arrangements of only some of a group of objects. *e.g. how many different four digit numbers can be made from the digits of* 12333210*?* *e.g. how many different numbers greater than* $20\,000$ *can be made from the digits of* 12333210*?*
7.01i 7.01j		i) Be able to solve problems about selections, including with constraints. *e.g. Find the number of ways in which a team of* 3 *men and* 2 *women can be selected from a group of* 6 *men and* 5 *women.*	j) Be able to solve problems with several constraints. *e.g. Given a graph showing who dislikes who, find the number of ways of choosing* 3 *men and* 2 *women from a group of* 4 *men and* 4 *women so that no two people chosen dislike each other.*

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
7.01k 7.01l 7.01m	The inclusion-exclusion principle	k) Be able to use the inclusion-exclusion principle for two sets in solving problems. *e.g.* $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. *Venn diagrams may be used.* *e.g. How many integers in* $\{1, 2, ..., 100\}$ *are not divisible by* 2 *or* 3*?*	l) Be able to extend the inclusion-exclusion principle to more than two sets. *e.g.* $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$ $- n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$. *Venn diagrams may be used.* *e.g. How many integers in* $\{1, 2, ..., 100\}$ *are not divisible by* 2*,* 3 *or* 5*?* m) Be able to find derangements. *Includes enumeration of the number of derangements of* n *objects,* D_n*, by ad hoc methods only.*
7.02 Graphs and Networks			
7.02a 7.02b 7.02c	Terminology and notation	a) Understand the meaning of the terms "vertex" (or "node") and "arc" (or "edge"). *Includes the concept of the "degree" of a vertex as the number of arcs "incident" to the vertex.* *Includes the term "adjacent" for pairs of vertices or edges.* b) Understand the meaning of the terms "tree", "simple", "connected" and "simply connected" as they refer to graphs. *Includes understanding and using the restrictions on the vertex degrees implied by these conditions.* c) Understand the meaning of the terms "walk", "trail", "path", "cycle" and "route". *A "walk" is a set of arcs where the end vertex of one is the start vertex of the next.* *A "trail" is a walk in which no arcs are repeated.* *A "path" is a trail in which no nodes are repeated.* *A "cycle" is a closed path.* *A "route" can be a walk, a trail or a path, or may be a closed walk, trail or path.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.02d	Complete graphs	d) Understand and be able to use the term “complete” and the notation K_n for a complete graph on n vertices. *Includes knowing that K_n has $\frac{1}{2}n(n-1)$ arcs.*	
7.02e 7.02f	Bipartite graphs	e) Understand and be able to use bipartite graphs and the notation $K_{m,n}$ for a complete bipartite graph connecting m vertices to n vertices. *Includes knowing that $K_{m,n}$ has mn arcs.*	f) Use a colouring argument to show that a given graph is, or is not, bipartite.
7.02g	Eulerian graphs	g) Use the degrees of vertices to determine whether a given graph is Eulerian, semi-Eulerian or neither. Understand what these terms mean in terms of traversing the graph.	
7.02h 7.02i	Hamiltonian graphs		h) Understand and be able to use the definition of a Hamiltonian path, a Hamiltonian cycle and a Hamiltonian graph. i) Know and use Ore’s theorem. *i.e. For a simple graph G with $n \geq 3$ vertices, if $\deg v + \deg w \geq n$ for every pair of distinct non-adjacent vertices v and w, then G is Hamiltonian.* *Includes understanding that Ore’s theorem gives a sufficient but not necessary condition for a graph to be Hamiltonian.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.02j	Isomorphism	j) Understand what it means to say that two graphs are isomorphic. Construct an isomorphism either by a reasoned argument or by explicit labelling of vertices. *Includes understanding that having the same degree sequence (ordered list of vertex degrees) is necessary but not sufficient to show isomorphism.* *Includes the term "non-isomorphic".*	
7.02k	Digraphs	k) Understand and be able to use digraphs. *Includes the terms "indegree" and "outdegree".*	
7.02l 7.02m 7.02n 7.02o	Planar graphs		l) Understand and be able to apply the concepts of planarity, subdivision and contraction. *i.e. Subdivision is inserting a vertex of degree 2 into an arc. Contraction is contracting two vertices into one so that any arc incident with the original two vertices is incident with the contracted vertex.* *Includes the notation (AB) for the vertex created by the contraction of the arc AB.* *Includes drawing planar representations.* m) Know, understand and use Euler's formula $\mathrm{V}+\mathrm{R}=\mathrm{E}+2$. n) Know and use Kuratowski's theorem. *i.e. A finite graph is planar if and only if it does not contain a subgraph that is a subdivision of K_5 or $K_{3,3}$.* o) Understand and be able to use the concept of thickness. [*Calculation of thickness > 2 is excluded.*]

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.02p 7.02q 7.02r	Using graphs and networks	p) Understand that a network is a weighted graph. Use graphs and networks to model the connections between objects. *Graphs and networks may be directed or undirected.* q) Use an adjacency matrix representation of a graph and a weighted matrix representation of a network. r) Be able to model problems using graphs or networks, and solve them.	
7.03 Algorithms			
7.03a	Definition of an algorithm	a) Understand that an algorithm has an input and an output, is deterministic and finite. *Includes the use of a counter and the use of a stopping condition in an algorithm.* *Be familiar with the terms "greedy", "heuristic" and "recursive" in the context of algorithms.*	
7.03b	Awareness of the uses and practical limitations of algorithms	b) Appreciate why an algorithmic approach to problem-solving is generally preferable to ad hoc methods, and understand the limitations of algorithmic methods. *Includes understanding that algorithmic methods are used by computers for solving large scale problems and that small scale problems are only being used to demonstrate how a given algorithm works.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.03c	Working with algorithms	c) Trace through an algorithm and interpret what the algorithm has achieved. Algorithms may be presented as flow diagrams, listed in words, or written in simple pseudo-code. *Includes understanding and being able to use the functions* $\text{INT}(x)$ *and* $\text{ABS}(x)$*. Learners may find it useful to have a calculator with these functions, but large numbers of repeated applications will not be required in the assessment.* *Includes adapting or altering an algorithm to achieve a given purpose, and adjusting a short set of instructions to create an algorithm.* [*Programming skills will not be required*.]	
7.03d	The order of an algorithm	d) Use the order of an algorithm to calculate an approximate run-time for a large problem by scaling up a given run-time. *Includes understanding that when the "maximum run-time" of an algorithm is represented as a function of the "size" of the problem, the order of the algorithm, for very large sized problems, is given by the dominant term.* *Learners should know that the sum of the first* n *positive integers is* $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$. *Learners should be familiar with the notation* $\text{O}(n^4)$ *and the concept of dominance in an informal sense only.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.03e	Efficiency and complexity	e) Compare the efficiency of two algorithms that achieve the same end result by considering a given aspect of the run-time in a specific case. *e.g. The number of swaps or comparisons to sort a given list.*	
7.03f 7.03h		f) Calculate worst case time complexity, the "maximum run-time" $\mathrm{T}(n)$, as a function of the size of a problem by considering the worst case for a specific problem. *Includes cases of the algorithms for sorting and standard network problems studied in this specification.* *Includes an informal understanding that, for example* $\mathrm{T}(n) = n^2 + n^4$ *is order* n^4*, or equivalently* $\mathrm{O}(n^4)$*.*	h) Calculate the run-time as a function of the size of a problem by considering the best case, the worst case or a typical case. *Includes considering all cases and averaging where appropriate.*
7.03g 7.03i		g) Be familiar with $\mathrm{O}(n^k)$, where *n* is a measure of the size of the problems and $k = 0,\ 1,\ 2,\ 3$ or 4.	i) Be familiar with: $\mathrm{O}(n^k)$ for $k \in \mathbb{R}$, $\mathrm{O}(a^n)$ for $a > 0$, $\mathrm{O}(\log n)$ where n is a measure of the size of the problem. *Know the hierarchy of orders and what this means in terms of efficiency. Learners should be aware that:* $\mathrm{O}(1) \subset \mathrm{O}(\log n) \subset \mathrm{O}(n) \subset \mathrm{O}(n\log n) \subset \mathrm{O}(n^2) \subset \mathrm{O}(n^3) \subset \ldots \subset \mathrm{O}(a^n) \subset \mathrm{O}(n!)$, *which is given in the Formulae Booklet.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.03j 7.03k	Strategies for sorting	j) Be able to sort a list using bubble sort and using shuttle sort. *Bubble sort and shuttle sort will start at the left-hand end of the list, unless specified otherwise in the question.* *Includes knowing that, in general, sorting algorithms have quadratic order as a function of the length of the list.*	k) Be able to sort a list using quick sort. *Quick sort will pivot on the first value, unless specified otherwise in a question.* *Includes knowing that quick sort is only* $\mathrm{O}(n^2)$ *in the worst case.* *Questions may be set that interrogate the application of the method, for example whether the choice of pivot affects the efficiency of quick sort.*
7.03l 7.03m	Strategies for packing	l) Be familiar with the next-fit, first-fit, first-fit decreasing and full bin methods for one-dimensional packing problems. *Includes knowing that these are heuristic algorithms.* *Includes the terms "online" and "offline".*	m) Extend their knowledge of packing methods. *e.g. Packing problems in two or three dimensions.* *e.g. Knapsack problems: given a set of items, each with a mass and a profit, determine which to include so that the total mass does not exceed some given limit and the total profit is as large as possible.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.04 Network Algorithms			
7.04a	Least weight path between two vertices	a) Be able to use examples to demonstrate understanding and use of Dijkstra's algorithm to find the length and route of a least weight (shortest) path. *Solve problems that require a least weight (shortest) path as part of their solution.* *Know that Dijkstra's algorithm has quadratic order (as a function of the number of vertices).*	
7.04b	Least weight set of arcs connecting all vertices	b) Be able to use examples to demonstrate understanding and use of Prim's algorithm (both in graphical and tabular/matrix form) and Kruskal's algorithm to find a minimum connector (minimum spanning tree) for a network. *Solve problems that require a minimum spanning tree as part of their solution.* *Includes adapting a solution to deal with practical issues.* *Know that Prim's algorithm and Kruskal's algorithm have cubic order (as a function of the number of vertices).*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.04c 7.04d	Least weight cycle through all vertices		c) Be able to use the nearest neighbour method on a network formed by weighting a complete graph to find an upper bound for the travelling salesperson problem. *Includes understanding that when the nearest neighbour method is used on a network formed by weighting a graph that is not complete it may stall before reaching every vertex or it may reach every vertex but to close the route it may need a path that is not a direct connection from the end vertex back to the start vertex.* *Includes choosing between two, or more, upper bounds to find the best (least) upper bound.* *Includes using short-cuts where possible to improve an upper bound.* d) Be able to use a minimum spanning tree on a reduced network to calculate a lower bound for the travelling salesperson problem on a complete graph and understand why this method gives a lower bound. *Understand that for a graph that is not complete this method can give a value that is not a lower bound.* *Includes choosing between two or more lower bounds to find the best (greatest) lower bound.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.04e	Least weight route through all vertices that traverses every arc at least once		e) Be able to solve the route inspection problem by consideration of all possible pairings of up to six odd nodes. *If a problem has more than six odd nodes additional restrictions will reduce the number of pairings that need to be considered.* *Problems may be set that require an understanding of how the number of pairings increases as the number of odd nodes increases. For* $2n$ *odd nodes the number of pairings is* $1 \times 3 \times \ldots \times (2n-1) = \prod_{r=1}^{n}(2r-1)$.
7.04f	Network problems	f) Be able to choose an appropriate algorithm to solve a practical problem. *Includes adapting an algorithm or a solution to deal with practical issues.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.05 Decision Making in Project Management			
7.05a	Critical path analysis	a) Be able to construct and interpret activity networks using activity on arc. *Appreciate that a path of critical activities (a critical path) is a longest path in a directed network.*	
7.05b		b) Be able to carry out a forward pass to determine earliest start times and find the minimum project completion time, and to carry out a backward pass to determine latest finish times and find the critical activities. *Includes understanding and using the terms "burst" and "merge".*	
7.05c 7.05d		c) Understand, and be able to calculate, (total) float.	d) Be able to find latest start times and earliest finish times for activities. Calculate and interpret independent and interfering float.
7.05e			e) Be able to use an activity network to construct a cascade chart and use a cascade chart to determine the effect on the minimum project completion time of a delay to one or more activities, or other scheduling restrictions. *Cascade charts may be constructed either with one activity on each row or with the critical activities together in one row and a row for each non-critical activity.* *Float may be shown using dashed lines.* *Includes constructing a schedule to show how a given number of workers can complete a project subject to given constraints.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.06 Graphical Linear Programming			
7.06a 7.06b	Formulating LP problems	a) Be able to set up a linear programming formulation in the form “maximise (or minimise) objective subject to inequality constraints, and trivial constraints of the form variable ≥ 0”. *Includes:* *1. identifying relevant variables, including units when appropriate,* *2. formulating constraints in these variables, including when the information is given in ratio form,* *3. writing down an objective function and stating whether it is to be maximised or minimised.*	b) Be able to use slack variables to convert inequality constraints, each being $\leq$ a non-negative constant, into equations, together with further trivial constraints of the form variable ≥ 0.
7.06c	Working with constraints	c) Be able to investigate constraints and objectives in numerical cases using algebra and ad hoc methods.	
7.06d 7.06e 7.06f	Graphical solutions	d) Be able to carry out and interpret a graphical solution for problems where the objective is a function of two variables, including cases where integer solutions are required. *The region where each inequality is not satisfied will be shaded, leaving the feasible region as the unshaded part of the graph.*	e) Be able to discuss the effect of making a change to one or two of the coefficients and how this will change the solution. f) Be able to carry out integer programming, including using the branch-and-bound method.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.07 The Simplex Algorithm			
7.07a	Use a simplex tableau		a) Be able to set up an initial simplex tableau in standard format. *Rows to show objective to be maximised, followed by constraints. Columns to represent objective, variables and then slack variables, with the column representing the right-hand side as the last column.*
7.07b			b) Be able to perform an iteration of a simplex tableau, including the choice of the pivot. *The column with the most negative value in the objective row should be chosen, unless other specific instruction is given.*
7.07c			c) Be able to interpret the values of the variables, slack variables and objective at any stage and know when the optimum has been achieved. *Includes discussing the effect of changes to the coefficients.*
7.07d	Terminology		d) Be able to use the terms "basic feasible solution", "basic variable" and "non-basic variable" appropriately. *Basic variables correspond to columns consisting of zeroes together with a single 1, the other variables are non-basic.*
7.07e	Graphical and algebraic interpretation of iterations		e) Understand what an iteration of the simplex algorithm achieves, in terms of moving along edges of a multi-dimensional convex polygon, usually in two or three dimensions, and be able to apply this to problems.
7.07f			f) Be able to explain, algebraically, some of the calculations used in the simplex algorithm.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
7.08 Game Theory			
7.08a 7.08b	Pay-off matrix	a) Understand the idea of a zero-sum game and its representation by means of a pay-off matrix. *Includes converting a game to a zero-sum form, where appropriate.* b) Be able to reduce a matrix using a dominance argument.	
7.08c 7.08d	Pure strategies	c) Be able to identify play-safe strategies and stable solutions and understand what they represent.	d) Be able to identify a Nash Equilibrium solution and understand what it represents.
7.08e 7.08f	Mixed strategies	e) Be able determine an optimal mixed strategy for a game with no stable solution by reducing to two variables and using simultaneous equations or a graphical method, where possible. *Includes knowing that the optimum may occur at an extreme value* ($p = 0$ *or* $p = 1$).	f) Be able to determine an optimal mixed strategy for a game with no stable solution by reformulating the problem as a linear programming problem that could be solved using the simplex algorithm.

2g. Content of Additional Pure Mathematics (Optional paper Y545)

Introduction to Additional Pure Mathematics.

In **Additional Pure Mathematics** learners will broaden and deepen their knowledge of pure mathematics, studying both discrete and continuous topics which form the foundation of undergraduate study in mathematics and mathematical disciplines. This area covers recurrence relations, number theory, group theory, the vector product, surfaces and partial differentiation.

8.01 Sequences and Series
Recurrence relations are explored, including their long term behaviour and solution of first and second order recurrence relations.

8.02 Number Theory
Number theory is introduced through number bases, modular arithmetic, divisibility algorithms and solving linear congruences. This foundation is extended to simultaneous linear congruences and Fermat's little theorem.

8.03 Groups
Group axioms and examples of finite groups of small order are studied, including cyclic groups. Lagrange's theorem is applied to the order of subgroups and the concept of an isomorphism is introduced.

8.04 Further Vectors
The concept of vector product introduced in the Pure Core is developed and extended to the scalar triple product, finding areas and volumes, and solving problems using vector methods.

8.05 Surfaces and Partial Differentiation
Surfaces in 3-D are explored, including contours and sections, and applying partial differentiation to find stationary points, and to classify them using the Hessian matrix.

8.06 Further Calculus
The calculus techniques in the Pure Core are extended to finding arc-lengths and surface areas of revolution and the use of reduction formulae.

Assumed knowledge
Learners are assumed to know the content of GCSE (9–1) Mathematics and A Level Mathematics. They are also assumed to know the content of Pure Core (Y540 and Y541). All of this content is assumed, but will only be explicitly assessed where it appears in this section.

Use of technology

To support the teaching and learning of mathematics using technology, we suggest that the following activities are carried out through the course:

1. Graphing tools: Learners could use graphing software to investigate the relationships between graphical and algebraic representations, including 3-D plots of surfaces and solids of revolution.

2. Computer Algebra System (CAS): Learners could use CAS software to investigate algebraic relationships, including manipulation of matrices, evaluating integrals, solving recurrence systems and solving equations and as an investigative problem solving tool. This is best done in conjunction with other software such as graphing tools and spreadsheets.

3. Visualisation: Learners could use appropriate software to visualise situations in 3-D relating to surfaces, and to linear transformations and invariance.

4. Spreadsheets: Learners could use spreadsheet software to investigate sequences and series, for modelling and to generate tables of values for functions.

Content of Additional Pure Mathematics (Optional paper Y545)

When this course is being co-taught with AS Level Further Mathematics A (H235) the 'Stage 1' column indicates the common content between the two specifications and the 'Stage 2' column indicates content which is particular to this specification. In each section the OCR reference lists 'Stage 1' statements before 'Stage 2' statements.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.01 Sequences and Series			
8.01a 8.01b	Recurrence relations	a) Be able to work with general sequences given either as recurrence relations or by position-to-term (closed form) formulae $u_n = \mathrm{f}(n)$. *The notation $\{u_n\}$ for sequences, which may or may not include a zeroth term, should be recognised.* b) Use induction to prove results relating to both sequences and series.	
8.01c 8.01d	Properties of sequences	c) Understand and be able to describe various possibilities for the behaviour of sequences. *Learners are expected to be able to use the terms "periodic", "convergence", "divergence", "oscillating", "monotonic".* *Note that a periodic sequence with period two may be referred to as "oscillating", but that both convergent and divergent sequences can oscillate. "Divergence" can refer to sequences that are bounded or unbounded.* d) Identify and be able to use the limit of the nth term of a sequence as $n \to \infty$, including steady-states. *Includes forming sequences from other sequences, for example, finding differences or ratios of successive terms of a sequence.* [*Rates of convergence are excluded.*]	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.01e	Fibonacci and related numbers	e) Be able to work with the Fibonacci numbers (and other Fibonacci-like sequences, such as the Lucas numbers), and understand their properties. *Includes recognising and using the properties of ϕ, both numerical and algebraic, and its role in the Fibonacci sequence.*	
8.01f 8.01g	Solving recurrence systems	f) Be able to solve a first-order linear recurrence relation with constant coefficients, using the associated auxiliary equation and complementary function. *Includes finding both general and particular solutions.* *Includes homogeneous and non-homogeneous recurrence relations of the form $u_{n+1} = au_n + f(n)$, where $f(n)$ may be a polynomial function or of the form dk^n.* *Includes knowing the terms, "closed form" and "position-to-term".* *Includes understanding that a "recurrence system" consists of a "recurrence relation", an "initial condition" and the range of the variable n.*	g) Be able to solve a second-order linear recurrence relation with constant coefficients, using the associated auxiliary equation and complementary function. *Includes finding both general and particular solutions.* *Includes the cases when the roots of the auxiliary equation are:* *(i) distinct and real,* *(ii) repeated,* *(iii) complex.* *Includes homogeneous and non-homogeneous recurrence relations of the form $u_{n+2} = au_{n+1} + bu_n + f(n)$, where $f(n)$ may be a polynomial function or of the form dk^n.* *Learners should be aware that this topic is the discrete analogue of the work on differential equations in H245 section 4.10.*
8.01h 8.01i	Modelling	h) Be able to apply their knowledge of recurrence relations to modelling. *Includes birth- and/or death-rates and the use of the $\text{INT}(x)$ function for discrete models. Learners may find it useful to have a calculator with this function, but large numbers of repeated applications will not be required in the assessment.*	i) Be able to extend their knowledge of modelling to second order recurrence relations.

OCR Ref.	Subject Content	Stage 1 learners should...	Stage 2 learners should additionally...
8.02 Number Theory			
8.02a	Number bases	a) Understand and be able to work with numbers written in base n, where n is a positive integer. *The standard notation for number bases will be used.* *i.e.* 2013_n *will denote the number* $2n^3 + n + 3$ *(with* $n > 3$ *in this example) and the letters* A*-*F *will be used to represent the integers* 10*-*15 *respectively when* $11 \le n \le 16$*.*	
8.02b	Divisibility tests	b) Be able to use (without proof) the standard tests for divisibility by $2, 3, 4, 5, 8, 9$ and 11. *Includes knowing that repeated tests can be used to establish divisibility by composite numbers.*	
8.02c		c) Be able to establish suitable (algorithmic) tests for divisibility by other primes less than 50. *For integers* a *and* b*, the notation* $a \mid b$ will be used for *"*a *divides exactly into* b*" ("*a *is a factor of* b*", "*b *is a multiple of* a*", etc.).*	
8.02d	The division algorithm	d) Appreciate that, for any pair of positive integers a, b with $0 < b \le a$, we can uniquely express a as $a = bq + r$ where q (the quotient) and r (the residue, or remainder, when a is divided by b) are both positive integers and $r < b$.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.02e 8.02g 8.02f 8.02h	Finite (modular) arithmetics	e) Understand and be able to use finite arithmetics (the arithmetic of integers modulo n for $n \geq 2$). f) Be able to solve single linear congruences of the form $ax \equiv b \pmod{n}$.	g) Be able to calculate quadratic residues and solve, or prove insoluble, equations involving them. h) Be able to solve simultaneous linear congruences of the form $ax \equiv b \pmod{n}$. *No more than three simultaneous linear congruences will be used. Use of the Chinese remainder theorem will be allowed but not required.*
8.02i 8.02j	Prime numbers	i) Understand the concepts of prime numbers, composite numbers, highest common factors (hcf), and coprimality (relative primeness). *Knowledge of the fundamental theorem of arithmetic will be expected, but proof of the result will not be required.* j) Know and be able to apply the result that $a \mid b$ and $a \mid c \Rightarrow a \mid (bx + cy)$ for any integers x and y. *Includes using this result, for example to test for common factors or coprimality.*	
8.02k	Euclid's lemma	k) Know and be able to use Euclid's lemma: if $a \mid rs$ and $\text{hcf}(a, r) = 1$ then $a \mid s$.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.02l	Fermat's little theorem		l) Know and be able to use Fermat's little theorem in both forms: 1. if p is prime and hcf $(a, p) = 1$, then $a^{p-1} \equiv 1 (\text{mod } p)$; 2. if p is prime $a^p \equiv a\,(\text{mod } p)$. *Includes recognising that if p is prime then Fermat's little theorem holds, but that the converse is not true (that is, be aware of the existence of "pseudo-primes" to base p).* [*The proof is excluded.*] [*Carmichael numbers are excluded.*]
8.02m 8.02n	The order of *a* modulo p		m) Know and be able to use the fact that $p - 1$ is not necessarily the least positive integer n for which $a^n \equiv 1 (\text{mod } p)$. n) Know that the n with this property, called "the order of a modulo p", is a factor of $p - 1$ and be able to find it in specific cases.
8.02o	Binomial theorem		o) Be able to use the binomial theorem to show that $(a + b)^p \equiv a^p + b^p (\text{mod } p)$ for prime p, and use this result.
8.03 Groups			
8.03a 8.03b	Binary operations	a) Be able to work with binary operations and their properties when defined on given sets. *Includes knowing and understanding the terms "associativity" and "commutativity".* b) Be able to construct Cayley tables for given finite sets under the action of a given binary operation. *Multiplicative notation and/or terminology will generally be used, when appropriate.*	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.03c	Definition of a group	c) Recall and be able to use the definition of a group, for example to show that a given structure is, or is not, a group. *e.g. Questions may be set on groups of integers modulo n (for $n \geq 2$), functions, matrices, transformations, the symmetries of given geometrical shapes and complex numbers.* *Groups may be referred to in either of the forms:* *1. by the given set and associated binary operation $(G, \circ)$,* *2. as "G", where the operation is understood, or* *3. as "the set G with the operation $\circ$".* *To include knowing the meaning of the terms "identity" and "closed", and that in an abelian group the operation is commutative.*	
8.03d		d) Recognise and be able to use the Latin square property for group tables.	
8.03e	Orders of elements and groups	e) Recall the meaning of the term "order", as applied both to groups and to elements of a group, and be able to determine the orders of elements in a given group. *Includes knowing and being able to use the fact that the order of an element is a factor of the order of the group.*	
8.03f	Subgroups	f) Understand and be able to use the definition of a subgroup, find subgroups and show that given subsets are, or are not, proper subgroups.	
8.03g	Cyclic groups	g) Recall the meaning of the term "cyclic" as applied to groups.	

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.03h	Generators	h) Understand that a cyclic group is generated by "powers" of a single element (generator), that there may be more than one such element within a group, and that other (non-cyclic) groups may be generated by two or more elements along with their "powers" and "products".	
8.03i 8.03j	Properties of groups	i) Be familiar with the structure of finite groups up to, and including, order seven, and be able to apply this knowledge in solving problems.	j) Be able to work with groups of higher finite order, or of infinite order. *No particular prior knowledge of specific groups, or their structures, will be expected.*
8.03k	Lagrange's theorem		k) Recall and be able to apply Lagrange's theorem concerning the order of a subgroup of a finite group. *The proof of the theorem is not required, but a clear statement of the result may be expected.*
8.03l	Isomorphism		l) Be able to determine whether two given groups are, or are not, isomorphic using informal methods. *e.g. By noting disparities between the orders of elements.*
8.03m	Abstract groups		m) Be able to work with groups defined by their algebraic properties. *Includes using algebraic methods to establish properties in abstract groups.* *e.g. To show that any group in which every element is self-inverse is abelian, to establish an identity or complete a Cayley table.*

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.04 Further Vectors			
8.04a	Vector product	a) Understand and be able to use the definition, in geometrical terms, of the vector product and be able to form the vector product in magnitude and direction, and in component form. *Includes use of the formula* $\mathbf{a}\times\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\hat{\mathbf{n}}$*, where* $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$*, in that order (and the vectors* $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$*, in that order) form a right-handed triple.*	
8.04b		b) Understand the anti-commutative and distributive properties of the vector product.	
8.04c 8.04e		c) Be able to use the vector product to calculate areas of triangles and parallelograms.	e) Understand and be able to use the definition of the scalar triple product, and be able to use it to calculate volumes of tetrahedra and parallelepipeds. *Includes the notation* $\mathbf{a}.(\mathbf{b}\times\mathbf{c})$*.* *Includes understanding the significance of* $\mathbf{a}.(\mathbf{b}\times\mathbf{c})=\mathbf{0}$*.*
8.04d		d) Understand the significance of $\mathbf{a}\times\mathbf{b}=\mathbf{0}$. *e.g. The equation of a line in the form* $(\mathbf{r}-\mathbf{a})\times\mathbf{d}=\mathbf{0}$*.*	
8.05 Surfaces and Partial Differentiation			
8.05a 8.05b	3-D surfaces	a) Be able to work with functions of two variables, given either explicitly in the form $z=\mathrm{f}(x,y)$ or implicitly in the form $\mathrm{g}(x,y,z)=c$, and understand and use the fact that this equation, and its partial derivatives, relate to a 3-D surface. *An informal understanding only of how the partial derivatives relate to the surface is required.* *Functions* $\mathrm{f}(x,y)$ *will involve sums and products of powers of* x *and* y *only. Issues relating to domains and ranges will not be considered beyond the appreciation that, for example the surface* $z=\frac{x}{y}$ *has no point at which* $y=0$*.*	b) Extend their knowledge of surfaces to those defined by functions of more than two variables, and incorporating trigonometric functions, logarithms and exponentials.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.05c	Sections and contours	c) Be able to sketch sections and contours, and know how these are related to the surface. *i.e. Sections of the form* $z = \mathrm{f}(a, y)$ *or* $z = \mathrm{f}(x, b)$ *and contours of the form* $c = \mathrm{f}(x, y)$.	
8.05d	Partial differentiation	d) Be able to find first and second derivatives, including mixed derivatives. *Learners will be expected to recognise and use both notations for first- and second-order partial derivatives, including mixed ones.* *e g.* $\frac{\partial \mathrm{f}}{\partial x}, \frac{\partial^2 \mathrm{f}}{\partial x \partial y}$ *and* $\mathrm{f}_x, \mathrm{f}_{xy}$. *Includes the Mixed derivative theorem; namely, that* $\frac{\partial^2 \mathrm{f}}{\partial x \partial y} = \frac{\partial^2 \mathrm{f}}{\partial y \partial x}$ *or* $\mathrm{f}_{xy} = \mathrm{f}_{yx}$ *for suitably well-defined, continuous functions* f.	
8.05e 8.05f	Stationary points	e) Understand and be able to apply the concept that stationary points of z arise when $\frac{\partial \mathrm{f}}{\partial x} = \frac{\partial \mathrm{f}}{\partial y} = 0$ (or $\mathrm{f}_x = \mathrm{f}_y = 0$) and that these can be maxima, minima or saddle-points. *Learners should know and understand the basic properties of these stationary points.* *Learners will only be required to find stationary points, but will not be required to determine their natures.*	f) Be able to determine, for 3-D surfaces given in the form $z = \mathrm{f}(x, y)$, the nature of maxima, minima and saddle-points by means of the sign of the determinant of the matrix of second partial derivatives (the Hessian Matrix), $\mathbf{H} = \begin{pmatrix} \mathrm{f}_{xx} & \mathrm{f}_{xy} \\ \mathrm{f}_{yx} & \mathrm{f}_{yy} \end{pmatrix}$; namely, that: 1. if $\lvert \mathbf{H} \rvert > 0$ and $\mathrm{f}_{xx} > 0$, there is a (local) minimum; 2. if $\lvert \mathbf{H} \rvert > 0$ and $\mathrm{f}_{xx} < 0$, there is a (local) maximum; 3. if $\lvert \mathbf{H} \rvert < 0$ there is a saddle-point; 4. if $\lvert \mathbf{H} \rvert = 0$ then the nature of the stationary point cannot be determined by this test.

OCR Ref.	Subject Content	Stage 1 learners should…	Stage 2 learners should additionally…
8.05g	Tangent planes		g) Be able to determine, for 3-D surfaces given in the form $z = \mathrm{f}(x, y)$, the equation of a tangent plane to the curve at a given point $(x, y, z) = (a, b, \mathrm{f}(a, b))$ using the formula $z = \mathrm{f}(a,b) + (x-a)\mathrm{f}_x(a, b) + (y-b)\mathrm{f}_y(a,b)$.
8.06 Further Calculus			
8.06a	Reduction formulae		a) Be able to establish and use given reduction formulae, and employ them to evaluate integrals using recursive techniques.
8.06b	Arc lengths and surface areas		b) Be able to find arc lengths and areas of surface of revolution for curves defined in cartesian or parametric form.

2h. Prior knowledge, learning and progression

- It is assumed that learners are familiar with the content of GCSE (9–1) Mathematics for first teaching from 2015 and the content of GCE A Level Mathematics for first teaching from September 2017.

- OCR's A Level in Further Mathematics A is designed for learners who wish to study beyond an A Level in Mathematics, and provides a solid foundation for progression into further study particularly in mathematics, engineering, computer science, the sciences and economics.

- OCR's A Level in Further Mathematics A is both broader and deeper than A Level Mathematics. A Level Further Mathematics builds from GCSE (9–1) Mathematics and A Level Mathematics. As well as building on the algebra and calculus introduced in A Level Mathematics, the A Level Further Mathematics pure core content introduces complex numbers and matrices; fundamental mathematical ideas with wide applications in mathematics, engineering, physical sciences and computing. The non-core content includes different options that can enable learners to specialise in areas of mathematics that are particularly relevant to their interests and future aspirations. A Level Further Mathematics prepares learners for further study and employment in highly mathematical disciplines that require knowledge and understanding of sophisticated mathematical ideas and techniques.

- Some learners may wish to follow a further mathematics course only up to AS Level, in order to broaden their curriculum, and to develop their interest and understanding of different areas of the subject.

- AS Level Further Mathematics A can be co-taught with A Level Further Mathematics A as a separate qualification. It consolidates and develops GCSE (9–1) Mathematics and AS Level Mathematics and supports transition to higher education or employment in any of the many disciplines that make use of quantitative analysis, including the social sciences, business, accounting and finance, mathematics, engineering, computer science, the sciences and economics.

There are a number of Mathematics specifications at OCR. Find out more at www.ocr.org.uk

3 Assessment of A Level in Further Mathematics A

3a. Forms of assessment

OCR's A Level in Further Mathematics A consists of two mandatory papers and two optional papers all of which are externally assessed.

OCR's A Level in Further Mathematics A is a linear qualification in which all papers must be taken in the same examination series.

Mandatory Pure Core

All learners will study the content of Pure Core. This Pure Core is assessed through two equal papers and synoptically within the optional papers as appropriate.

Optional papers

Learners will study at least two areas chosen from Statistics, Mechanics, Discrete Mathematics and Additional Pure Mathematics. Each area is assessed in a single paper.

All six papers (Y540–Y545) contain assessment of the Overarching Themes, some extended response questions and some stretch and challenge questions.

Stretch and challenge questions are designed to allow the most able learners the opportunity to demonstrate the full extent of their knowledge and skills.

Stretch and challenge questions will support the awarding of A* grade at A Level, addressing the need for greater differentiation between the most able learners.

Any valid combination of four papers will include at least one unstructured problem solving question, which addresses multiple areas of the problem solving cycle as set out in the Overarching Themes in section 2b.

Any valid combination of four papers will include at least two complete problem solving questions, which address the first two bullet points of Assessment Objective 3 in combination and at least two complete modelling questions, which address the last three bullet points of Assessment Objective 3 in combination. See section 3b.

All examinations have a duration of 90 minutes.

Allowable calculators can be used for any function they can perform.

In each question paper, learners are expected to support their answers with appropriate working.

See section 2b for use of calculators.

Pure Core 1 (Y540)
This paper is worth 25% of the total A Level. All questions are compulsory and there are 75 marks in total.

The paper assesses content from the Pure Core section of the specification, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Pure Core 2 (Y541)
This paper is worth 25% of the total A Level. All questions are compulsory and there are 75 marks in total.

The paper assesses content from the Pure Core section of the specification, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Statistics (Y542)
This paper is worth 25% of the total A Level. All questions are compulsory and there are 75 marks in total.

The paper assesses content from the Statistics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Mechanics (Y543)
This paper is worth 25% of the total A level. All questions are compulsory and there are 75 marks in total.

The paper assesses content from the Mechanics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Discrete Mathematics (Y544)
This paper is worth 25% of the total A Level. All questions are compulsory and there are 75 marks in total.

The paper assesses content from the Discrete Mathematics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

Additional Pure Mathematics (Y545)
This paper is worth 25% of the total A Level. All questions are compulsory and there are 75 marks in total.

The paper assesses content from the Additional Pure Mathematics section of the specification, and synoptically from the Pure Core, in the context of all of the Overarching Themes.

The assessment has an increasing gradient of difficulty through the paper and consists of a mix of short and long questions.

3b. Assessment Objectives (AO)

There are 3 Assessment Objectives in OCR A Level in Further Mathematics A. These are detailed in the table below.

	Assessment Objectives	Weightings A Level
AO1	**Use and apply standard techniques** Learners should be able to: • select and correctly carry out routine procedures; and • accurately recall facts, terminology and definitions.	50%
AO2	**Reason, interpret and communicate mathematically** Learners should be able to: • construct rigorous mathematical arguments (including proofs); • make deductions and inferences; • assess the validity of mathematical arguments; • explain their reasoning; and • use mathematical language and notation correctly. *Where questions/tasks targeting this assessment objective will also credit learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'solve problems within mathematics and other contexts' (AO3) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s).*	At least 15% for any valid combination of papers
AO3	**Solve problems within mathematics and in other contexts** Learners should be able to: • translate problems in mathematical and non-mathematical contexts into mathematical processes; • interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations; • translate situations in context into mathematical models; • use mathematical models; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them. *Where questions/tasks targeting this assessment objective will also credit learners for the ability to 'use and apply standard techniques' (AO1) and/or to 'reason, interpret and communicate mathematically' (AO2) an appropriate proportion of the marks for the question/task will be attributed to the corresponding assessment objective(s).*	At least 15% for any valid combination of papers

3

AO weightings in A Level in Further Mathematics

The target number of marks allocated to each AO for each paper, out of a total of 75 for each paper, is given in the table below.

The number of marks for AO2 and AO3 reflects the appropriate balance of assessment for the content in each paper.

Paper	Number of marks		
	AO1	AO2	AO3
Pure Core 1 (Y540)	37–38 marks	18–19 marks	18–19 marks
Pure Core 2 (Y541)	37–38 marks	18–19 marks	18–19 marks
Statistics (Y542)	37–38 marks	18–19 marks	18–19 marks
Mechanics (Y543)	37–38 marks	10–11 marks	26–27 marks
Discrete Mathematics (Y544)	37–38 marks	18–19 marks	18–19 marks
Additional Pure Mathematics (Y545)	37–38 marks	22–23 marks	14–15 marks

Each set of assessments which constitutes a valid combination will achieve the weighting shown in the following table.

Each of these sets of assessment gives an overall percentage of 50% (±2%) for AO1 and at least 15% for each of AO2 and AO3 as required.

Pure Core 1 + Pure Core 2 + choice of two options	% of overall A Level in Further Mathematics A (H245)		
	AO1	AO2	AO3
Statistics and Mechanics	148–152 marks	64–68 marks	80–84 marks
Statistics and Discrete Mathematics	148–152 marks	72–76 marks	72–76 marks
Statistics and Additional Pure Mathematics	148–152 marks	76–80 marks	68–72 marks
Mechanics and Discrete Mathematics	148–152 marks	64–68 marks	80–84 marks
Mechanics and Additional Pure Mathematics	148–152 marks	68–72 marks	76–80 marks
Discrete Mathematics and Additional Pure Mathematics	148–152 marks	76–80 marks	68–72 marks
Total	50% (±2%)	At least 15%	At least 15%

3c. Total qualification time

Total qualification time (TQT) is the total amount of time, in hours, expected to be spent by a learner to achieve a qualification. It includes both guided learning hours and hours spent in preparation, study, and assessment. The total qualification time for A Level in Further Mathematics A is 360 hours. The total guided learning time is 360 hours.

3d. Qualification availability outside of England

This qualification is available in England. For Wales and Northern Ireland please check the Qualifications in Wales Portal (QIW) or the Northern Ireland Department of Education Performance Measures / Northern Ireland Entitlement Framework Qualifications Accreditation Number (NIEFQAN) list to see current availability.

3e. Language

This qualification is available in English only. All assessment materials are available in English only and all candidate work must be in English.

3f. Assessment availability

There will be one examination series available each year in May/June to **all** learners.

All examined papers must be taken in the same examination series at the end of the course.
This specification will be certificated from the June 2019 examination series onwards.

3g. Retaking the qualification

Learners can retake the qualification as many times as they wish. They must retake a complete valid combination of papers for the qualification as detailed in section 2a.

3h. Assessment of extended response

The assessment materials for this qualification provide learners with the opportunity to demonstrate their ability to construct and develop a sustained and coherent line of reasoning and marks for extended responses are integrated into the marking criteria. Tasks which offer this opportunity will be found across all six papers.

3i. Synoptic assessment

Mathematics is, by nature, a synoptic subject. The assessment in this specification allows learners to demonstrate the understanding they have acquired from the course as a whole and their ability to integrate and apply that understanding. This level of understanding is needed for successful use of the knowledge and skills from this course in future life, work and study.

In the examination papers, learners will be required to integrate and apply their understanding in order to address problems which require both breadth and depth of understanding in order to reach a satisfactory solution.

Learners will be expected to reflect on and interpret solutions, drawing on their understanding of different aspects of the course.

Tasks which offer this opportunity will be found in all papers.

3j. Calculating qualification results

A learner's overall qualification grade for A Level in Further Mathematics A will be calculated by adding together their marks from the four papers taken to give their total raw mark. This mark will then be compared to the qualification level grade boundaries that apply for the combination of papers taken by the learner and for the relevant exam series to determine the learner's overall qualification grade.

Where learners take more than the required number of optional papers, the combination of papers that result in the best grade will be used.

Note: this may NOT be the combination with the highest number of raw marks.

The total raw mark will be the total from the combination that leads to the best grade.

4 Admin: what you need to know

The information in this section is designed to give an overview of the processes involved in administering this qualification so that you can speak to your exams officer. All of the following processes require you to submit something to OCR by a specific deadline.

More information about the processes and deadlines involved at each stage of the assessment cycle can be found in the Administration area of the OCR website. OCR's *Admin overview* is available on the OCR website at http://www.ocr.org.uk/administration.

4a. Pre-assessment

Estimated entries

Estimated entries are your best projection of the number of learners who will be entered for a qualification in a particular series. Estimated entries should be submitted to OCR by the specified deadline. They are free and do not commit your centre in any way.

Final entries

Final entries provide OCR with detailed data for each learner, showing each assessment to be taken. It is essential that you use the correct entry code, considering the relevant entry rules.

Final entries must be submitted to OCR by the published deadlines or late entry fees will apply.

All learners taking an A Level in Further Mathematics must be entered for H245.

All learners must also be entered for each of the papers they are taking using the relevant entry codes.

Paper entry codes are given in the table below.

All learners take Y540 and Y541 and **at least** two of the optional papers Y542, Y543, Y544 and Y545 to be awarded OCR's A Level in Further Mathematics A.

Where learners take more than the required number of optional papers, the combination of papers that result in the best grade will be used.

Note: this may NOT be the combination with the highest number of raw marks.

The total raw mark will be the total from the combination that leads to the best grade.

Entry code	Title	Paper code	Paper title	Assessment type
H245	Further Mathematics A	Y540	Pure Core 1	External Assessment (Mandatory)
		Y541	Pure Core 2	External Assessment (Mandatory)
		Y542	Statistics	External Assessment (Optional)
		Y543	Mechanics	External Assessment (Optional)
		Y544	Discrete Mathematics	External Assessment (Optional)
		Y545	Additional Pure Mathematics	External Assessment (Optional)

Collecting evidence of student performance to ensure resilience in the qualifications system

Regulators have published guidance on collecting evidence of student performance as part of long-term contingency arrangements to improve the resilience of the qualifications system. You should review and consider this guidance when delivering this qualification to students at your centre.

For more detailed information on collecting evidence of student performance please visit our website at: https://www.ocr.org.uk/administration/general-qualifications/assessment/

4b. Special consideration

Special consideration is a post–assessment adjustment to marks or grades to reflect temporary injury, illness or other indisposition at the time the assessment was taken.

Detailed information about eligibility for special consideration can be found in the JCQ publication *A guide to the special consideration process.*

4

4c. External assessment arrangements

Regulations governing examination arrangements are contained in the JCQ *Instructions for conducting examinations*.

Head of centre annual declaration

The Head of Centre is required to provide a declaration to the JCQ as part of the annual NCN update, conducted in the autumn term, to confirm that the centre is meeting all of the requirements detailed in the specification. Any failure by a centre to provide the Head of Centre Annual Declaration will result in your centre status being suspended and could lead to the withdrawal of our approval for you to operate as a centre.

Private candidates

Private candidates may enter for OCR assessments.

A private candidate is someone who pursues a course of study independently but takes an examination or assessment at an approved examination centre. A private candidate may be a part-time student, someone taking a distance learning course, or someone being tutored privately. They must be based in the UK.

Private candidates need to contact OCR approved centres to establish whether they are prepared to host them as a private candidate. The centre may charge for this facility and OCR recommends that the arrangement is made early in the course.

Further guidance for private candidates may be found on the OCR website: https://www.ocr.org.uk

4d. Results and certificates

Grade Scale

A Level qualifications are graded on the scale: A*, A, B, C, D, E, where A* is the highest. Learners who fail to reach the minimum standard for E will be Unclassified (U). Only subjects in which grades A* to E are attained will be recorded on certificates.

Papers are graded on the scale a*, a, b, c, d, e, where a* is the highest. Learners who fail to reach the minimum standard for e with be unclassified (u). Individual paper results will not be recorded on certificates.

Results

Results are released to centres and learners for information and to allow any queries to be resolved before certificates are issued.

Centres will have access to the following results information for each learner:

- the grade for the qualification
- the raw mark for each paper
- the total raw mark for the qualification.

The following supporting information will be available:

- raw mark grade boundaries for each paper
- raw mark grade boundaries for all combinations of papers.

Until certificates are issued, results are deemed to be provisional and may be subject to amendment.

A learner's final results will be recorded on an OCR certificate.

The qualification title will be shown on the certificate as 'OCR Level 3 Advanced GCE in Further Mathematics A'.

4

4e. Post-results services

A number of post-results services are available:

- **Review of marking** – If you are not happy with the outcome of a learner's results, centres may request a review of marking. Full details of the post-results services are provided on the OCR website.
- **Missing and incomplete results** – This service should be used if an individual subject result for a learner is missing, or the learner has been omitted entirely from the results supplied.
- **Access to scripts** – Centres can request access to marked scripts.

4f. Malpractice

Any breach of the regulations for the conduct of examinations and non-exam assessment work may constitute malpractice (which includes maladministration) and must be reported to OCR as soon as it is detected.

Detailed information on malpractice can be found in the JCQ publication *Suspected Malpractice in Examinations and Assessments: Policies and Procedures*.

5 Appendices

5a. Overlap with other qualifications

This qualification overlaps with AS Further Mathematics A and with other specifications in A Level Further Mathematics and AS Further Mathematics.

5b. Accessibility

Reasonable adjustments and access arrangements allow learners with special educational needs, disabilities or temporary injuries to access the assessment and show what they know and can do, without changing the demands of the assessment. Applications for these should be made before the examination series. Detailed information about eligibility for access arrangements can be found in the JCQ *Access Arrangements and Reasonable Adjustments.*

The A Level qualification and subject criteria have been reviewed in order to identify any feature which could disadvantage learners who share a protected Characteristic as defined by the Equality Act 2010. All reasonable steps have been taken to minimise any such disadvantage.

5

5c. Mathematical notation

The tables below set out the notation that must be used by AS and A Level Mathematics and Further Mathematics specifications. Learners will be expected to understand this notation without need for further explanation. Any additional notation required is listed in the relevant content statement in section 2 of the specification.

1	Set Notation	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x : \ldots\}$	the set of all x such that $\ldots$
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$

1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	$(x,\ y)$	the ordered pair x, y
1.20	$[a,\ b]$	the closed interval $\{x \in \mathbb{R} : a \le x \le b\}$
1.21	$[a,\ b)$	the interval $\{x \in \mathbb{R} : a \le x < b\}$
1.22	$(a,\ b]$	the interval $\{x \in \mathbb{R} : a < x \le b\}$
1.23	$(a,\ b)$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
1.24	$\mathbb{C}$	the set of complex numbers
2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \le$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \ge$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence

2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence
3	**Operations**	
3.1	$a+b$	a plus b
3.2	$a-b$	a minus b
3.3	$a \times b,\ ab,\ a.b$	a multiplied by b
3.4	$a \div b,\ \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1,\ n \in \mathbb{N};\ 0! = 1$
3.10	$\binom{n}{r}, {}^nC_r, {}_nC_r$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x,\ \delta x$	an increment of x
4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x),\ \mathrm{f}''(x),\ \ldots,\ \mathrm{f}^{(n)}(x)$	the first, second, ..., n^{th} derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x},\ \ddot{x},\ \ldots$	the first, second, ... derivatives of x with respect to t

5

4.11	$\int y \, dx$	the indefinite integral of y with respect to x
4.12	$\int_a^b y \, dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_e x$	natural logarithm of x
6	**Trigonometric and Hyperbolic Functions**	
6.1	$\left.\begin{array}{l}\sin, \cos, \tan \\ \operatorname{cosec}, \sec, \cot\end{array}\right\}$	the trigonometric functions
6.2	$\left.\begin{array}{l}\sin^{-1}, \cos^{-1}, \tan^{-1} \\ \arcsin, \arccos, \arctan\end{array}\right\}$	the inverse trigonometric functions
6.3	$°$	degrees
6.4	rad	radians
6.5	$\left.\begin{array}{l}\operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot}\end{array}\right\}$	the inverse trigonometric functions
6.6	$\left.\begin{array}{l}\sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \coth\end{array}\right\}$	the hyperbolic functions
6.7	$\left.\begin{array}{l}\sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \coth^{-1}\end{array}\right\}$ $\left.\begin{array}{l}\operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth}\end{array}\right\}$	the inverse hyperbolic functions
7	**Complex Numbers**	
7.1	i, j	square root of -1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\mathrm{Re}(z)$	the real part of z, $\mathrm{Re}(z) = x$
7.6	$\mathrm{Im}(z)$	the imaginary part of z, $\mathrm{Im}(z) = y$
7.7	$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$

7.8	$\arg(z)$	the argument of z, $\arg(z)=\theta, -\pi<\theta\le\pi$
7.9	z^*	the complex conjugate of z, $x-\mathrm{i}y$
8	**Matrices**	
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.6	Δ, det $\mathbf{M}$ or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix $\mathbf{M}$
8.7	$\mathbf{Mr}$	image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert\mathbf{a}\rvert, a$	the magnitude of $\mathbf{a}$
9.6	$\lvert\overrightarrow{\mathrm{AB}}\rvert$, AB	the magnitude of $\overrightarrow{\mathrm{AB}}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i}+b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector
9.12	$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
10	**Differential Equations**	
10.1	ω	angular speed
11	**Probability and Statistics**	
11.1	A, B, C, etc.	events
11.2	$A \cup B$	union of the events A and B

5

11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.6	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
11.7	X, Y, R, etc.	random variables
11.8	x, y, r, etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X = x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim \mathrm{N}(0, 1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0, 1)$
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	Null hypothesis
11.29	H_1	Alternative hypothesis
11.30	r	product-moment correlation coefficient for a sample
11.31	ρ	product-moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres

12.4	m/s, m s^{-1}	metres per second (velocity)
12.5	m/s^2, m s^{-2}	metres per second per second (acceleration)
12.6	F	force or resultant force
12.7	N	Newton
12.8	N m	Newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction

5d. Mathematical formulae and identities

Learners must be able to use the following formulae and identities for A Level Further Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms.

These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$$a^x a^y \equiv a^{x+y}$$

$$a^x \div a^y \equiv a^{x-y}$$

$$(a^x)^y \equiv a^{xy}$$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a (x^k)$

5

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression

$u_n = a + (n-1)d$

General term of a geometric progression

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area $= \dfrac{1}{2} ab \sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and area of circle, radius r and diameter d

$C = 2\pi r = \pi d \qquad A = \pi r^2$

Pythagoras' theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse

$c^2 = a^2 + b^2$

Area of a trapezium $= \dfrac{1}{2}(a+b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation

Volume of a prism = area of cross section × length

For a circle of radius r, where an angle at the centre of θ radians subtends an arc of length l and encloses an associated sector of area a

$l = r\theta \qquad a = \dfrac{1}{2} r^2 \theta$

Complex Numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

Loci in the Argand diagram

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis

Exponential Form

$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

For $ax^2 + bx + c = 0$ with roots α and β:

$$\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$$

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ :

$$\sum \alpha = \frac{-b}{a} \qquad \sum \alpha\beta = \frac{c}{a} \quad \alpha\beta\gamma = \frac{-d}{a}$$

Hyperbolic Functions

$$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$$

$$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$$

$$\tanh x \equiv \frac{\sinh x}{\cosh x}$$

5

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\sinh kx$	$k\cosh kx$
$\cosh kx$	$k\sinh kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
$\cosh kx$	$\frac{1}{k}\sinh kx+c$
$\sinh kx$	$\frac{1}{k}\cosh kx+c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c,\ x\neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $= \int_a^b y\,dx\,(y\geq 0)$

Volumes of revolution about the x and y axes

$V_x=\pi\int_a^b y^2\,dx$ $\qquad$ $V_y=\pi\int_c^d x^2\,dy$

Simple Harmonic Motion

$\ddot{x}=-\omega^2 x$

Vectors

$|x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$

$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$$

where θ is the angle between the vectors **a** and **b**

The equation of the line through the point with position vector **a** parallel to vector **b** is

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

The equation of the plane containing the point with position vector **a** and perpendicular to vector **n** is

$(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$

5

Mechanics

Forces and Equilibrium

Weight = mass $\times g$

Friction: $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration

$v = \dfrac{\mathrm{d}r}{\mathrm{d}t} \qquad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$

$r = \int v \, \mathrm{d}t \qquad v = \int a \, \mathrm{d}t$

$v = \dfrac{\mathrm{d}s}{\mathrm{d}t} \qquad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

$s = \int v \, \mathrm{d}t \qquad v = \int a \, \mathrm{d}t$

Statistics

The mean of a set of data: $\bar{x} = \dfrac{\sum x}{n} = \dfrac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \dfrac{X - \mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$

Learners will be given the following formulae in the Formulae Booklet in each assessment.

Pure Mathematics

Arithmetic series

$$S_n=\frac{1}{2}n(a+l)=\frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$S_\infty=\frac{a}{1-r} \text{ for } |r|<1$$

Binomial series

$$(a+b)^n=a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+\dots+{}^nC_ra^{n-r}b^r+\dots+b^n \quad (n\in\mathbb{N}),$$

where $${}^nC_r={}_nC_r=\binom{n}{r}=\frac{n!}{r!(n-r)!}$$

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\dots+\frac{n(n-1)\dots(n-r+1)}{r!}x^r+\dots \quad (|x|<1, n\in\mathbb{R})$$

Series

$$\sum_{r=1}^{n}r^2=\frac{1}{6}n(n+1)(2n+1), \quad \sum_{r=1}^{n}r^3=\frac{1}{4}n^2(n+1)^2$$

Maclaurin series

$$\mathrm{f}(x)=\mathrm{f}(0)+\mathrm{f}'(0)x+\frac{\mathrm{f}''(0)}{2!}x^2+\dots+\frac{\mathrm{f}^{(r)}(0)}{r!}x^r+\dots$$

$$\mathrm{e}^x=\exp(x)=1+x+\frac{x^2}{2!}+\dots+\frac{x^r}{r!}+\dots \text{ for all } x$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\dots+(-1)^{r+1}\frac{x^r}{r}+\dots(-1<x\le 1)$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots+(-1)^r\frac{x^{2r+1}}{(2r+1)!}+\dots \text{ for all } x$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\dots+(-1)^r\frac{x^{2r}}{(2r)!}+\dots \text{ for all } x$$

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\dots+\frac{n(n-1)\dots(n-r+1)}{r!}x^r+\dots \quad (|x|<1, n\in\mathbb{R})$$

Matrix transformations

Reflection in the line $y=\pm x$: $\begin{pmatrix} 0 & \pm1 \\ \pm1 & 0 \end{pmatrix}$

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Rotations through θ about the coordinate axes. The direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation.

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\mathbf{R}_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Differentiation

f(x)	f′ (x)
tan kx	$k \sec^2 kx$
sec x	sec x tan x
cot x	$-\text{cosec}^2\, x$
cosec x	–cosec x cot x
arcsin x or $\sin^{-1}x$	$\dfrac{1}{\sqrt{1-x^2}}$
arccos x or $\cos^{-1}x$	$-\dfrac{1}{\sqrt{1-x^2}}$
arctan x or $\tan^{-1}x$	$\dfrac{1}{1+x^2}$

Quotient rule $y=\dfrac{u}{v}$, $\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx=\ln|f(x)|+c$$

$$\int f'(x)(f(x))^n\,dx=\frac{1}{n+1}(f(x))^{n+1}+c$$

5

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$

The mean value of $\mathrm{f}(x)$ on the interval $[a, b]$ is $\frac{1}{b-a}\int_a^b \mathrm{f}(x)\,\mathrm{d}x$

Area of sector enclosed by polar curve is $\frac{1}{2}\int r^2\,\mathrm{d}\theta$

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left(x+\sqrt{x^2+a^2}\right)$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\left(x+\sqrt{x^2-a^2}\right) \qquad (x>a)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h=\frac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1}=x_n-\frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Complex numbers

Circles: $\lvert z-a\rvert=k$

Half lines: $\arg(z-a)=\alpha$

Lines: $\lvert z-a\rvert=\lvert z-b\rvert$

De Moivre's theorem: $\{r(\cos\theta+\mathrm{i}\sin\theta)\}^n=r^n(\cos n\theta+\mathrm{i}\sin n\theta)$

Roots of unity: The roots of $z^n=1$ are given by $z=\exp\left(\frac{2\pi k}{n}\mathrm{i}\right)$ for $k=0, 1, 2, \ldots, n-1$

Vectors and 3-D coordinate geometry

Cartesian equation of the line through the point A with position vector $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ in direction $\mathbf{u}=u_1\mathbf{i}+u_2\mathbf{j}+u_3\mathbf{k}$ is $\frac{x-a_1}{u_1}=\frac{y-a_2}{u_2}=\frac{z-a_3}{u_3}(=\lambda)$

Cartesian equation of a plane is $n_1x+n_2y+n_3z+d=0$

Vector product: $\mathbf{a}\times\mathbf{b}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{vmatrix}\mathbf{i} & a_1 & b_1\\ \mathbf{j} & a_2 & b_2\\ \mathbf{k} & a_3 & b_3\end{vmatrix}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$

The distance between skew lines is $D = \frac{|(\mathbf{b}-\mathbf{a}).\mathbf{n}|}{|\mathbf{n}|}$, where **a** and **b** are position vectors of points on each line and **n** is a mutual perpendicular to both lines

The distance between a point and a line is $D = \frac{|ax_1 + by_1 - c|}{\sqrt{a^2 + b^2}}$, where the coordinates of the point are (x_1, y_1) and the equation of the line is given by $ax + by = c$

The distance between a point and a plane is $D = \frac{|\mathbf{b}.\mathbf{n} - p|}{|\mathbf{n}|}$, where **b** is the position vector of the point and the equation of the plane is given by $\mathbf{r}.\mathbf{n} = p$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1 - \frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is small and measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq (k + \frac{1}{2})\pi\right)$

Hyperbolic functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh^{-1} x = \ln[x + \sqrt{(x^2 + 1)}]$

$\cosh^{-1} x = \ln[x + \sqrt{(x^2 - 1)}], x \geq 1$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), -1 < x < 1$

Simple harmonic motion

$x = A\cos(\omega t) + B\sin(\omega t)$

$x = R\sin(\omega t + \varphi)$

Statistics

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\frac{\sum(x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$

5

Sampling distributions

For any variable X, $\mathrm{E}(\overline{X}) = \mu$, $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$ and $\overline{X}$ is approximately normally distributed when n is large enough (approximately $n > 25$)

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Unbiased estimates of the population mean and variance are given by $\frac{\sum x}{n}$ and $\frac{n}{n-1}\left(\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2\right)$

Expectation algebra

Use the following results, including the cases where $a = b = \pm 1$ and/or $c = 0$:

1. $\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$,
2. if X and Y are independent then $\mathrm{Var}(aX + bY + c) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$.

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with $\mathrm{P}(X = x_i) = p_i$

Expectation: $\mu = \mathrm{E}(X) = \sum x_i p_i$

Variance: $\sigma^2 = \mathrm{Var}(X) = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

	$P(X = x)$	$\mathrm{E}(X)$	$\mathrm{Var}(X)$
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Uniform distribution over $1, 2, \ldots, n$, $\mathrm{U}(n)$	$\frac{1}{n}$	$\frac{n+1}{2}$	$\frac{1}{12}(n^2-1)$
Geometric distribution $\mathrm{Geo}(p)$	$(1-p)^{x-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson $\mathrm{Po}(\lambda)$	$\mathrm{e}^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ

Continuous distributions

X is a continuous random variable with probability density function (p.d.f.) $\mathrm{f}(x)$

Expectation: $\mu = \mathrm{E}(X) = \int x\,\mathrm{f}(x)\,\mathrm{d}x$

Variance: $\sigma^2 = \mathrm{Var}(X) = \int (x - \mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$

Cumulative distribution function $\mathrm{F}(x) = \mathrm{P}(X \le x) = \int_{-\infty}^{x} \mathrm{f}(t)\,\mathrm{d}t$

	p.d.f.	E(X)	Var(X)
Continuous uniform distribution over $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \le z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

5

Non-parametric tests

Goodness-of-fit test and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_v^2$

Approximate distributions for large samples

Wilcoxon Signed Rank test: $T \sim N\left(\frac{1}{4}n(n+1), \frac{1}{24}n(n+1)(2n+1)\right)$

Wilcoxon Rank Sum test (samples of sizes m and n, with $m \le n$) :

$$W \sim N\left(\frac{1}{2}m(m+n+1), \frac{1}{12}mn(m+n+1)\right)$$

Correlation and regression

For a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}, \; S_{yy} = \sum(y_i - \bar{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n},$$

$$S_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

Product-moment correlation coefficient:

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left[\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(\sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}\right)\right]}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

Spearman's rank correlation coefficient: $r_s = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$

Critical values for the product moment correlation coefficient, r

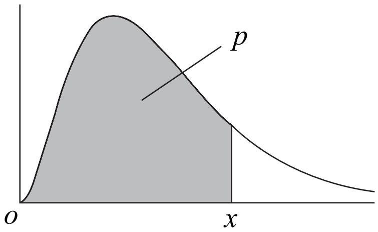
1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					*n*				
1	–	–	–	–	31	0.3009	0.3550	0.4158	0.4556
2	–	–	–	–	32	0.2960	0.3494	0.4093	0.4487
3	0.9877	0.9969	0.9995	0.9999	33	0.2913	0.3440	0.4032	0.4421
4	0.9000	0.9500	0.9800	0.9900	34	0.2869	0.3388	0.3972	0.4357
5	0.8054	0.8783	0.9343	0.9587	35	0.2826	0.3338	0.3916	0.4296
6	0.7293	0.8114	0.8822	0.9172	36	0.2785	0.3291	0.3862	0.4238
7	0.6694	0.7545	0.8329	0.8745	37	0.2746	0.3246	0.3810	0.4182
8	0.6215	0.7067	0.7887	0.8343	38	0.2709	0.3202	0.3760	0.4128
9	0.5822	0.6664	0.7498	0.7977	39	0.2673	0.3160	0.3712	0.4076
10	0.5494	0.6319	0.7155	0.7646	40	0.2638	0.3120	0.3665	0.4026
11	0.5214	0.6021	0.6851	0.7348	41	0.2605	0.3081	0.3621	0.3978
12	0.4973	0.5760	0.6581	0.7079	42	0.2573	0.3044	0.3578	0.3932
13	0.4762	0.5529	0.6339	0.6835	43	0.2542	0.3008	0.3536	0.3887
14	0.4575	0.5324	0.6120	0.6614	44	0.2512	0.2973	0.3496	0.3843
15	0.4409	0.5140	0.5923	0.6411	45	0.2483	0.2940	0.3457	0.3801
16	0.4259	0.4973	0.5742	0.6226	46	0.2455	0.2907	0.3420	0.3761
17	0.4124	0.4821	0.5577	0.6055	47	0.2429	0.2876	0.3384	0.3721
18	0.4000	0.4683	0.5425	0.5897	48	0.2403	0.2845	0.3348	0.3683
19	0.3887	0.4555	0.5285	0.5751	49	0.2377	0.2816	0.3314	0.3646
20	0.3783	0.4438	0.5155	0.5614	50	0.2353	0.2787	0.3281	0.3610
21	0.3687	0.4329	0.5034	0.5487	51	0.2329	0.2759	0.3249	0.3575
22	0.3598	0.4227	0.4921	0.5368	52	0.2306	0.2732	0.3218	0.3542
23	0.3515	0.4132	0.4815	0.5256	53	0.2284	0.2706	0.3188	0.3509
24	0.3438	0.4044	0.4716	0.5151	54	0.2262	0.2681	0.3158	0.3477
25	0.3365	0.3961	0.4622	0.5052	55	0.2241	0.2656	0.3129	0.3445
26	0.3297	0.3882	0.4534	0.4958	56	0.2221	0.2632	0.3102	0.3415
27	0.3233	0.3809	0.4451	0.4869	57	0.2201	0.2609	0.3074	0.3385
28	0.3172	0.3739	0.4372	0.4785	58	0.2181	0.2586	0.3048	0.3357
29	0.3115	0.3673	0.4297	0.4705	59	0.2162	0.2564	0.3022	0.3328
30	0.3061	0.3610	0.4226	0.4629	60	0.2144	0.2542	0.2997	0.3301

Critical values for Spearman's rank correlation coefficient, r_s

1-Tail Test	5%	2½%	1%	½%	1-Tail Test	5%	2½%	1%	½%
2-Tail Test	10%	5%	2%	1%	2-Tail Test	10%	5%	2%	1%
n					*n*				
1	–	–	–	–	31	0.3012	0.3560	0.4185	0.4593
2	–	–	–	–	32	0.2962	0.3504	0.4117	0.4523
3	–	–	–	–	33	0.2914	0.3449	0.4054	0.4455
4	1.0000	–	–	–	34	0.2871	0.3396	0.3995	0.4390
5	0.9000	1.0000	1.0000	–	35	0.2829	0.3347	0.3936	0.4328
6	0.8286	0.8857	0.9429	1.0000	36	0.2788	0.3300	0.3882	0.4268
7	0.7143	0.7857	0.8929	0.9286	37	0.2748	0.3253	0.3829	0.4211
8	0.6429	0.7381	0.8333	0.8810	38	0.2710	0.3209	0.3778	0.4155
9	0.6000	0.7000	0.7833	0.8333	39	0.2674	0.3168	0.3729	0.4103
10	0.5636	0.6485	0.7455	0.7939	40	0.2640	0.3128	0.3681	0.4051
11	0.5364	0.6182	0.7091	0.7545	41	0.2606	0.3087	0.3636	0.4002
12	0.5035	0.5874	0.6783	0.7273	42	0.2574	0.3051	0.3594	0.3955
13	0.4835	0.5604	0.6484	0.7033	43	0.2543	0.3014	0.3550	0.3908
14	0.4637	0.5385	0.6264	0.6791	44	0.2513	0.2978	0.3511	0.3865
15	0.4464	0.5214	0.6036	0.6536	45	0.2484	0.2945	0.3470	0.3822
16	0.4294	0.5029	0.5824	0.6353	46	0.2456	0.2913	0.3433	0.3781
17	0.4142	0.4877	0.5662	0.6176	47	0.2429	0.2880	0.3396	0.3741
18	0.4014	0.4716	0.5501	0.5996	48	0.2403	0.2850	0.3361	0.3702
19	0.3912	0.4596	0.5351	0.5842	49	0.2378	0.2820	0.3326	0.3664
20	0.3805	0.4466	0.5218	0.5699	50	0.2353	0.2791	0.3293	0.3628
21	0.3701	0.4364	0.5091	0.5558	51	0.2329	0.2764	0.3260	0.3592
22	0.3608	0.4252	0.4975	0.5438	52	0.2307	0.2736	0.3228	0.3558
23	0.3528	0.4160	0.4862	0.5316	53	0.2284	0.2710	0.3198	0.3524
24	0.3443	0.4070	0.4757	0.5209	54	0.2262	0.2685	0.3168	0.3492
25	0.3369	0.3977	0.4662	0.5108	55	0.2242	0.2659	0.3139	0.3460
26	0.3306	0.3901	0.4571	0.5009	56	0.2221	0.2636	0.3111	0.3429
27	0.3242	0.3828	0.4487	0.4915	57	0.2201	0.2612	0.3083	0.3400
28	0.3180	0.3755	0.4401	0.4828	58	0.2181	0.2589	0.3057	0.3370
29	0.3118	0.3685	0.4325	0.4749	59	0.2162	0.2567	0.3030	0.3342
30	0.3063	0.3624	0.4251	0.4670	60	0.2144	0.2545	0.3005	0.3314

Critical values for the χ^2 distribution

If X has a χ^2 distribution with v degrees of freedom then, for each pair of values of p and v, the table gives the value of x such that
$P(X \le x) = p$.



p	0.01	0.025	0.05	0.90	0.95	0.975	0.99	0.995	0.999
$v = 1$	$0.0^{3}1571$	$0.0^{3}9821$	$0.0^{2}3932$	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

Wilcoxon signed rank test

W_+ is the sum of the ranks corresponding to the positive differences,
W_- is the sum of the ranks corresponding to the negative differences,
T is the smaller of W_+ and W_-.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *T*

	Level of significance			
One Tail Two Tail	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n, each of W_+ and W_- can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

5

Wilcoxon rank sum test

The two samples have sizes m and n, where $m \le n$.
R_m is the sum of the ranks of the items in the sample of size m.
W is the smaller of R_m and $m(m+n+1)-R_m$.

For each pair of values of m and n, the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of W

	Level of significance											
One Tail	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two Tail	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One Tail	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two Tail	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n, the normal distribution with mean $\frac{1}{2}m(m+n+1)$ and variance $\frac{1}{12}mn(m+n+1)$ should be used as an approximation to the distribution of R_m.

Mechanics

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{v.v} = \mathbf{u.u} + 2\mathbf{a.s}$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Newton's experimental law

Between two smooth spheres $v_1 - v_2 = -e(u_1 - u_2)$

Between a smooth sphere with a fixed plane surface $v = -eu$

Motion in a circle

Tangential velocity is $v = r\dot{\theta}$

Radial acceleration is $\frac{v^2}{r}$ or $r\dot{\theta}^2$ towards the centre

Tangential acceleration is $\dot{v} = r\ddot{\theta}$

Centres of mass

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius r: $\frac{3}{8}r$ from centre

Hemispherical shell, radius r: $\frac{1}{2}r$ from centre

Circular arc, radius r, angle at centre 2α: $\frac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α: $\frac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\frac{1}{3}h$ above the base on the line from centre of base to vertex

5

Discrete Mathematics

Inclusion-exclusion principle

For sets A, B and C:

$n(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}) = n(\mathrm{A}) + n(\mathrm{B}) + n(\mathrm{C}) - n(\mathrm{A} \cap \mathrm{B}) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

The hierarchy of orders

$\mathrm{O}(1) \subset \mathrm{O}(\log n) \subset \mathrm{O}(n) \subset \mathrm{O}(n \log n) \subset \mathrm{O}(n^2) \subset \mathrm{O}(n^3) \subset \ldots \subset \mathrm{O}(a^n) \subset \mathrm{O}(n!)$

Sorting algorithms

Bubble sort:

Start at the left hand end of the list unless specified otherwise.

Compare the first and second values and swap if necessary. Then compare the (new) second value with the third value and swap if necessary. Continue in this way until all values have been considered.

Fix the last value then repeat with the reduced list until either there is a pass in which no swaps occur or the list is reduced to length 1, then stop.

5

Shuttle sort:

Start at the left hand end of the list unless specified otherwise.

Compare the second value with the first and swap if necessary, this completes the first pass. Next compare the third value with the second and swap if necessary, if a swap happened shuttle back to compare the (new) second with the first as in the first pass, this completes the second pass.

Next compare the fourth value with the third and swap if necessary, if a swap happened shuttle back to compare the (new) third value with the second as in the second pass (so if a swap happened shuttle back again). Continue in this way for $n-1$ passes, where n is the length of the list.

Quick sort:

The first value in any sublist will be the pivot, unless specified otherwise.

Working from left to right, write down each value that is smaller than the pivot, then the pivot, then work along the list and write down each value that is not smaller than the pivot. This produces two sublists (one of which may be empty) with the pivot between them and completes the pass.

Next apply this procedure to each of the sublists from the previous pass, unless they consist of a single entry, to produce further sublists. Continue in this way until no sublist has more than one entry.

Network algorithms

Dijkstra's algorithm

START with a graph *G*. At each vertex draw a box, the lower area for temporary labels, the upper left hand area for the order of becoming permanent and the upper right hand area for the permanent label.

STEP 1 Make the given start vertex permanent by giving it permanent label 0 and order label 1.

STEP 2 For each vertex that is not permanent and is connected by an arc to the vertex that has just been made permanent (with permanent label = P), add the arc weight to P. If this is smaller than the best temporary label at the vertex, write this value as the new best temporary label.

STEP 3 Choose the vertex that is not yet permanent which has the smallest best temporary label. If there is more than one such vertex, choose any one of them. Make this vertex permanent and assign it the next order label.

STEP 4 If every vertex is now permanent, or if the target vertex is permanent, use 'trace back' to find the routes or route, then STOP; otherwise return to STEP 2.

Prim's algorithm (graphical version)

START with an arbitrary vertex of *G*.

STEP 1 Add an edge of minimum weight joining a vertex already included to a vertex not already included.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Prim's algorithm (tabular version)

START with a table (or matrix) of weights for a connected weighted graph.

STEP 1 Cross through the entries in an arbitrary row, and mark the corresponding column.

STEP 2 Choose a minimum entry from the uncircled entries in the marked column(s).

STEP 3 If no such entry exists STOP; otherwise go to STEP 4.

STEP 4 Circle the weight w_{ij} found in STEP 2; mark column i; cross through row i.

STEP 5 Return to STEP 2.

Kruskal's algorithm

START with all the vertices of *G*, but no edges; list the edges in increasing order of weight.

STEP 1 Add an edge of *G* of minimum weight in such a way that no cycles are created.

STEP 2 If a spanning tree is obtained STOP; otherwise return to STEP 1.

Nearest neighbour method

START at a given vertex of *G*.

STEP 1 Find the least weight arc from this vertex to a vertex that has not already been included (or back to the start vertex if every vertex has been included).

STEP 2 If no such arc exists then the method has stalled STOP; otherwise add this arc to the path.

STEP 3 If a cycle has been found STOP; otherwise return to STEP 1.

Lower bound for travelling salesperson problem

START with all vertices and arcs of G.

STEP 1 Remove a given vertex and all arcs that are directly connected to that vertex, find a minimum spanning tree for the resulting reduced network.

STEP 2 Add the weight of this minimum connector to the sum of the two least weight arcs that had been deleted. This gives a lower bound.

Route inspection problem

START with a list of the odd degree vertices.

STEP 1 For each pair of odd nodes, find the connecting path of least weight.

STEP 2 Group the odd nodes so that the sum of weights of the connecting paths is minimised.

STEP 3 Add this sum to the total weight of the graph STOP.

The simplex algorithm

START with a tableau in standard format.

STEP 1 Choose a column with a negative entry in the objective row (or zero in degenerate cases).

STEP 2 The pivot row is the one for which non-negative value of the entry in the final column divided by the positive value of the entry in the pivot column is minimised. The pivot element is the entry of the pivot row in the chosen column.

STEP 3 Divide all entries in the pivot row by the value of the pivot element.

STEP 4 Add to, or subtract from, all other old rows a multiple of the new pivot row, so that the pivot column ends up consisting of zeroes and a single one, and corresponds to the new basic variable.

STEP 5 If the objective row has no negative entries STOP; otherwise return to STEP 1.

Additional Pure Mathematics

Vector product

$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta \hat{\mathbf{n}}$, where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order form a right-handed triple.

Surfaces

For 3-D surfaces given in the form $z = \mathrm{f}(x, y)$, the Hessian Matrix is given by $\mathbf{H} = \begin{pmatrix} \mathrm{f}_{xx} & \mathrm{f}_{xy} \\ \mathrm{f}_{yx} & \mathrm{f}_{yy} \end{pmatrix}$.

At a stationary point of the surface:

1. if $|\mathbf{H}| > 0$ and $\mathrm{f}_{xx} > 0$, there is a (local) minimum;
2. if $|\mathbf{H}| > 0$ and $\mathrm{f}_{xx} < 0$, there is a (local) maximum;
3. if $|\mathbf{H}| < 0$ there is a saddle-point;
4. if $|\mathbf{H}| = 0$ then the nature of the stationary point cannot be determined by this test.

The equation of a tangent plane to the curve at a given point $(x, y, z) = (a, b, \mathrm{f}(a, b))$ is
$z = \mathrm{f}(a, b) + (x - a)\mathrm{f}_x(a, b) + (y - b)\mathrm{f}_y(a, b)$.

Calculus

Arc length $s = \displaystyle\int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$

$$s = \int_a^b \sqrt{(\dot{x}^2 + \dot{y}^2)}\,\mathrm{d}t$$

Surface area of revolution $S_x = 2\pi \displaystyle\int_a^b y\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$ $\qquad S_y = 2\pi \displaystyle\int_c^d x\sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\,\mathrm{d}y$

$$S_x = 2\pi \int_a^b y(t)\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \quad S_y = 2\pi \int_c^d x(t)\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t$$

Summary of updates

Date	Version	Section	Title of section	Change
June 2018	1.1	Front cover	Disclaimer	Addition of Disclaimer
September 2019	1.2	Multiple 2b	 Content of A Level in Further Mathematics A (H245)	Correction of minor typographical errors Insertion of additional command words
April 2020	1.3	1d	How do I find out more information?	Insertion of Online Support Centre link
February 2021	1.4			Update to specification covers to meet digital accessibility standards
April 2022	1.5	2d 5d	Content of Statistics (Optional paper Y542) Mathematical formulae and identities	Correction of minor typographical errors
March 2023	1.6	3c	Total qualification time	Update to include total qualification time and guided learning hours (TQT/GLH) to comply with Qualifications in Wales regulations
February 2024	1.7	3d, 3e 4a Checklist	Qualification availability, Language Pre-assessment	Inclusion of disclaimer regarding language and availability Update to include resilience guidance Inclusion of Teach Cambridge

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